

SERGEI ZAGNY

MAGIC STARS

tables for piano or other instruments

VOLUME 1

1982-2008

N^Z
G_A
Y

Zagny Edition 2008

Score 018.1

© 2008 Sergei Zagny

I should like to express special thanks to Ivan Soshinsky,
who carefully read the Russian text of the Preface
and made many valuable suggestions,
and to Keith Hammond, who rendered great assistance
in preparing the English translation of the Preface.

Sergei Zagny

Выражаю особенную благодарность Ивану Сошинскому,
который внимательно прочитал русский текст предисловия
и дал множество ценных советов,
и Кису Хэммонду, который оказал неоценимую помощь
в переводе предисловия на английский язык.

Сергей Загний

СЕРГЕЙ ЗАГНИЙ

МАГИЧЕСКИЕ ЗВЁЗДЫ

таблицы для фортепиано или других инструментов

ТОМ 1

1982-2008

N ^Z G_Y

CONTENTS

VOLUME 1

Preface	VI
Предисловие	XIV
1. Vertical Stars	1.1-6
Vertical Stars Expanded	1.7-12
Vertical Stars Compressed	1.13-14
2. Horizontal Stars	2.1-6
Horizontal Stars Sloped	2.7-12
Horizontal Stars Extra Sloped	2.13-18
Horizontal Stars Compressed	2.19-20
Horizontal Stars Degenerated	2.21-22
3. Sides 1-4	3.1-2
Sides of Triangles 1-2	3.3
Sides of Downward/Upward Triangles	3.4
Opposite Sides 1-2	3.5-6
Inverse Sides 1-2	3.7-8
Interlinked Sides 1-2	3.9-12
Interlinked Sides Joined in Chains 1-2	3.13-14
Horizontal Sides 1-4	3.15-16
Horizontal Sides of Triangles 1-2	3.17
Horizontal Sides of Downward/Upward Triangles	3.18
Horizontal Opposite Sides 1-2	3.19-20
4. Angles	4.1-2
Crosses	4.3-4
Horizontal Angles 1-2	4.5-6
Horizontal Crosses 1-2	4.7-8
5. Parallel Sides and Complements	5.1-2
Complements 1-2	5.3-4
Odd Complements 1-4	5.5-6
Even Complements 1-4	5.7-8
Horizontal Parallel Sides and Complements	5.9-10
Horizontal Complements 1-2	5.11-12
6. Outer Triads 1-2	6.1-2
Inner Triads 1-2	6.3-4
Outer and Inner Triads 1-2	6.5-6
Outer and Inner Equilateral Triangles 1-2	6.7-8
Horizontal Triads 1-2	6.9-10
Horizontal Equilateral Triangles 1-2	6.11-12
7. Outer Hexads 1-4	7.1-2
Inner Hexads 1-4	7.3-4
Outer and Inner Hexads 1-2	7.5-6
Outer and Inner Hexads Expanded 1-2	7.7-10
Hexads Sorted by Rotations 1-2	7.11-14
Reversible Outer Hexads 1-2	7.15-16
Reversible Inner Hexads 1-2	7.17-18
Horizontal Hexads 1-2	7.19-20
8. Enneads	8.1-4
Odd Enneads Sorted by Rotations	8.5-8
Even Enneads Sorted by Rotations	8.9-12
Horizontal Enneads 1-2	8.13-14

9.	Dodecads	9.1-4
	Horizontal Dodecads 1-2	9.5-6
10.	Opposite Vertices	10.1
	Horizontal Opposite Vertices	10.2

VOLUME 2

11.	Stars in Base-n Mode	11.1-12
12.	Ordered Sides 1-12	12.1-12
	Ordered Horizontal Sides 1-12	12.13-24
	Sides Sorted by Structure	12.25
	Horizontal Sides Sorted by Structure	12.26
13.	Ordered Even Complements 1-8	13.1-8
	Ordered Odd Complements 1-8	13.9-16
	Complements Sorted by Structure	13.17-18
	Even Complements Sorted by Structure	13.19
	Odd Complements Sorted by Structure	13.20
14.	Ordered Outer Triads 1-6	14.1-6
	Ordered Inner Triads 1-6	14.7-12
	Outer Triads Sorted by Structure 1-2	14.13-14
	Inner Triads Sorted by Structure 1-2	14.15-16
15.	Ordered Outer Hexads 1-12	15.1-12
	Ordered Inner Hexads 1-12	15.13-24
	Outer Hexads Sorted by Structure	15.25
	Inner Hexads Sorted by Structure	15.26
16.	Ordered Enneads 1-6	16.1-12
	Enneads Sorted by Structure	16.13-16
17.	Ordered Dodecads 1-2	17.1-4
	Dodecads Sorted by Structure	17.5-8
18.	All 4-Note Combinations that Make the Sum of 26	18.1
	All 4-Note Combinations Sorted by Structure	18.2
	All 3-Note Combinations that Make the Sum of 13/26	18.3
	All 3-Note Combinations Sorted by Structure	18.4
	All 6-Note Combinations that Make the Sum of 26/52	18.5
	All 6-Note Combinations Sorted by Structure	18.6
	All 9-Note Combinations that Make the Sum of 65	18.7
	All 9-Note Combinations Sorted by Structure	18.8
19.	Vertical Antistars	19.1-4
20.	Horizontal Antistars	20.1-4
	Horizontal Antistars Expanded	20.5-8

Appendix.

Twelve Poems	A.1-6
--------------------	-------

PREFACE

This score is a series of musical tables that represent magic stars and their components in different forms and combinations. Notes correspond to values of which the stars consist, note combinations correspond to shapes. Audition time may be likened to the time during which the shapes are contemplated. In the score note durations are not defined: the contemplation time may vary.

◊ ◊ ◊

A *Magic Star* is a six-pointed star with numbers from 1 to 12 placed on six corner points (*outer vertices*) and six cross points (*inner vertices*). The star has six *sides*, each of which contains four vertices (two outer and two inner). The values are arranged so that their sum is the same for each side as well as for the six outer vertices. It always equals 26. (Since the total of all vertices is $1+2+\dots+12=78$, the number of sides is 6 and each value belongs to two sides, therefore $78 : 6 \times 2 = 26$.) There are a total of six different magic stars, disregarding variations produced by rotations and reflections. Each star is assigned a certain ordinal number.

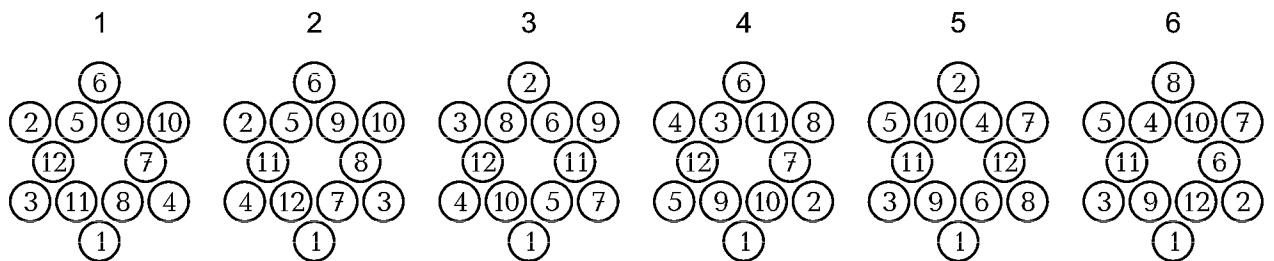


Fig. 1: Magic Stars, horizontal form

We distinguish two main forms of a star: *horizontal*, when two of six sides of a star are disposed horizontally (Fig. 1), and *vertical*, when two of six sides are disposed vertically (Fig. 2).

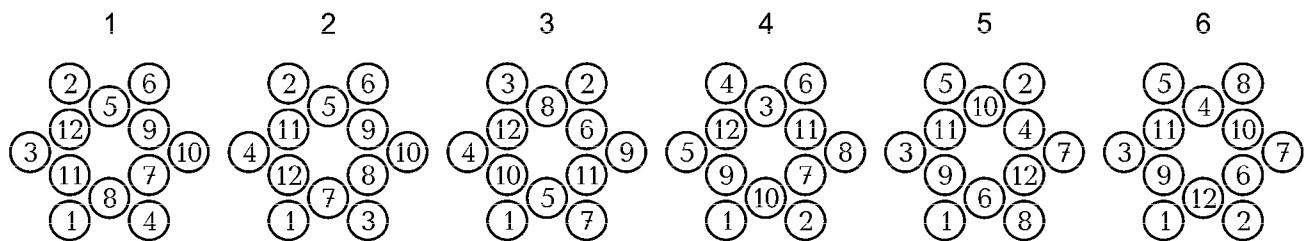


Fig. 2: Magic Stars, vertical form

Each horizontal and each vertical star has 12 variations. The variation in which '1' is placed at the bottom of a star and the value of the top left vertex is smaller than the value of the top right vertex is considered the first variation (or the base form) of a horizontal star (variation I or IL) (Fig. 1). Variation I rotated by 1/6 of a circle counter-clockwise gives variation II or IL; rotated by 2/6 it becomes variation III or IIII, etc. Variations I, II, ..., VI reflected relative to the vertical axis give six more variations which are IR, IIR, ..., VIR respectively (Fig. 3). Variations of a vertical star are named similarly (I, IR, II, IIR...). Vertical variations I, II, ..., VI come out of rotating the respec-

tive horizontal variations by 1/12 of a circle clockwise. These variations reflected relative to the vertical axis give variations IR, IIR, ..., VIR.

Vertices are also assigned ordinal numbers. A vertex having the value '1' is always the first, then the numbers sequentially increase: clockwise for variations I-VI and counter-clockwise for variations IR-VIR (Fig. 3).

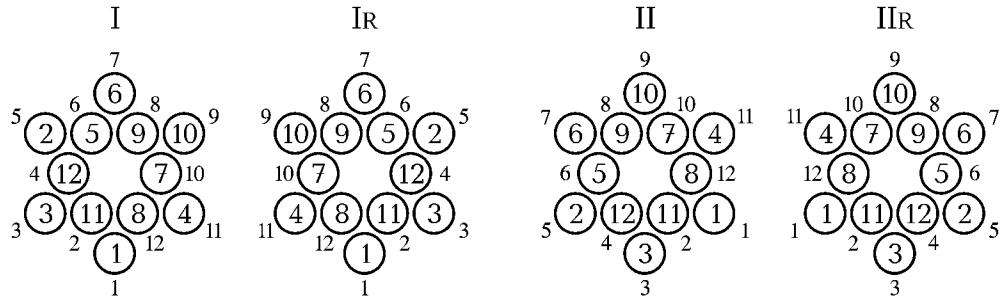


Fig. 3: Horizontal variations I, IR, II, IIR of the first star;
figures near cells are the numbers of the vertices

Note that the numbers of outer vertices are always odd and those of inner vertices are always even.

Any star may be represented as $\{n_i\}$, where n is a value and i is a number of a vertex (Fig. 4).

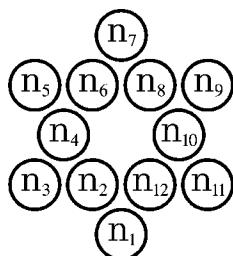


Fig. 4

In the stars we distinguish the following components:

- sides** (tetrachords);
- complements** (tetrachords or diamonds),
odd and even;
- triads** (trichords or triangles),
outer and inner;
- equilateral triangles**,
outer and inner;
- hexads** (hexachords of hexagons),
outer and inner;
- enneads** (enneachords of large triangles);
- dodecads** (dodecachords of dodecagons).

We also distinguish the following combinations:

- parallel sides;**
- opposite sides;**
- angles;**
- crosses;**
- opposite vertices.**

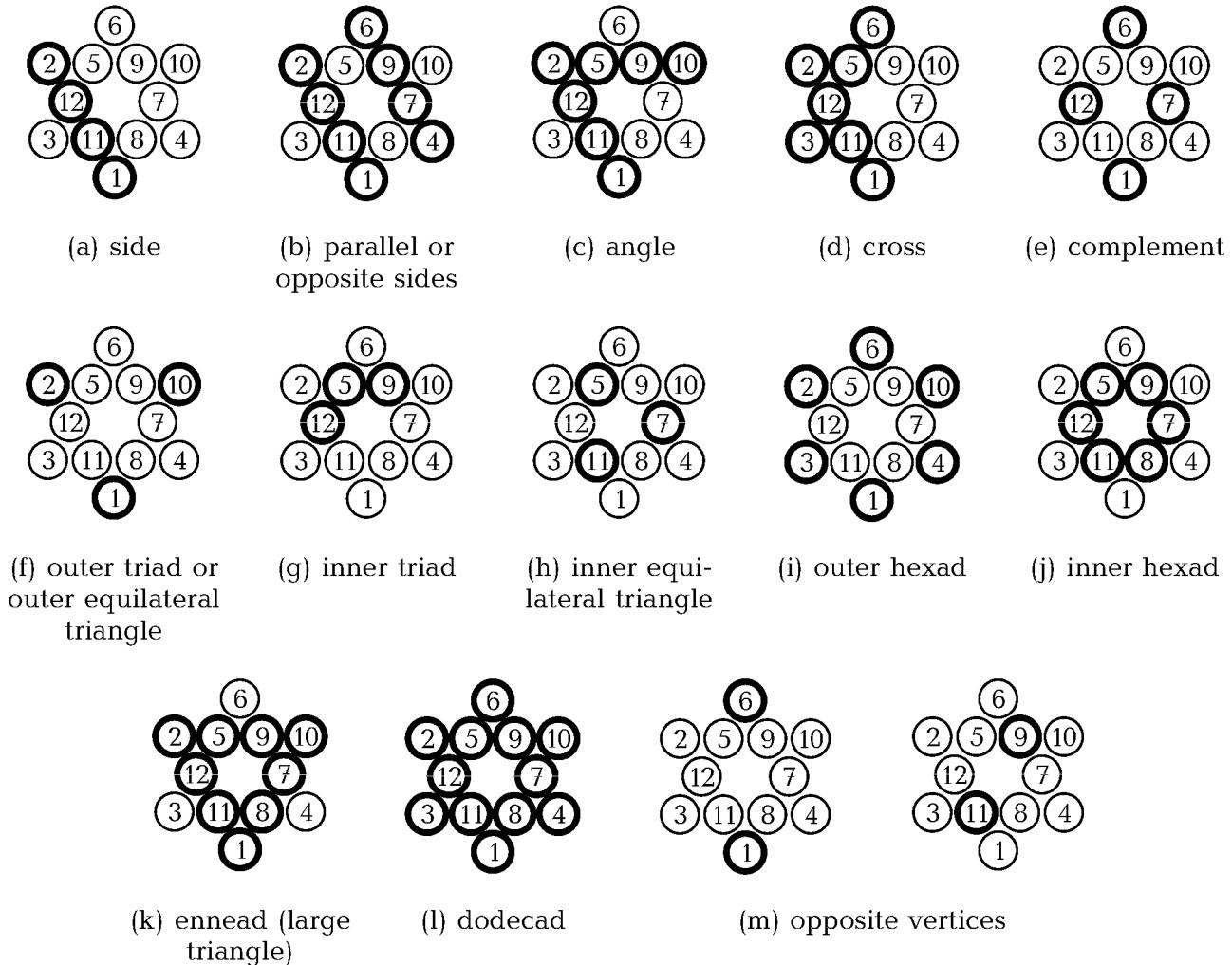


Fig. 5: Components and combinations

A *side* of a star (which is also a side of one of the two large triangles) is a segment having four vertices (Fig. 5a). The outer vertices are on the ends of a segment, the inner vertices are inside the segment. There are 6 sides in total in a star and the sum of the vertex values of any side is 26. Each side has 2 variations, for example, $\langle 1, 11, 12, 2 \rangle$ and $\langle 2, 12, 11, 1 \rangle$, in which the vertices are read in the counter direction.

Parallel sides are sides that have no common vertices (Fig. 5b). Each side in a star has exactly one parallel side. Parallel sides have eight vertices, their sum being $26 \times 2 = 52$. If parallel sides are read in the counter direction, they are called *opposite sides*.

An *angle* is two intersecting sides with one common outer vertex (Fig. 5c). A *cross* is two intersecting sides with one common inner vertex. (Fig. 5d) Either an angle or a cross consists on seven vertices and their sum is variable. However, if an intersecting vertex is counted twice, the sum is always $26 \times 2 = 52$.

If any two parallel sides are removed from a star, four vertices arranged in the shape of a diamond remain. These vertices are called the *complement* (Fig. 5e). There are three complements in a star. The sum of the vertex values of each complement is 26. The vertices in the variations of the complement are read in a different order. Eight variations are presented for each complement in the score. There are 8 variations for each complement represented in the score. Four start and end with outer (odd) vertices and they are called *odd complements*. The four remaining variations start and end with inner (even) vertices and are called *even complements*.

A *triad* is three outer vertices giving the sum of the sum of 26 : 2=13 which is an *outer triad* (Fig. 5f), or three inner vertices giving the sum of 26 which is an *inner triad* (Fig. 5g). There are four triads in a star, two outer and two inner. For each star they are well defined. One of the outer triads always encloses vertex '1'; one of the inner triads always encloses vertex '12'. An outer triad always consists of three angular vertices of a large equilateral triangle (i.e. an outer triad always coincides with an *outer equilateral triangle*, Fig. 5f). Conversely, the vertices of an inner triad never form an equilateral triangle and the vertices forming an *inner equilateral triangle* never have the sum of 26 (Fig. 5h). Each triad and each equilateral triangle has 6 variations. Vertices are read clockwise in three of them and counterclockwise in the three others.

A *hexad* is six outer vertices, which is an *outer hexad* (Fig. 5i), or six inner vertices, which is an *inner hexad* (Fig. 5j). The sum of the vertex values of an outer hexad is 26, the sum of the vertex values of an inner hexad is 52. A star has two hexads, one outer and one inner. Each hexad has 12 variations. Vertices are read clockwise in six of these and counterclockwise in the six others.

An *ennead* (or *large triangle*) comprises three outer vertices (one of two outer triads) and six inner ones (Fig. 5k). The sum of the vertex values of an ennead is $13 + 52 = 65$. A star has two enneads. In the score each is represented by 12 variations (in which an outer vertex is either the first or the last). Vertices are read clockwise in six variations and counterclockwise in the six others.

A *dodecad* comprise all the vertices of a star (Fig. 5l). The sum of the vertex values of a dodecad is 78. There is one dodecad in a star. It has 24 variations. Vertices are read clockwise in twelve variations and counterclockwise in the twelve others.

◊ ◊ *

Values that form stars have been translated into music in the following way: '1' corresponds to *E*, '2' corresponds to *F* and so on, up the chromatic scale.

◊ * ◊

Where the components (sides, complements etc.) are examined separately from the stars, these components are always accompanied by indices. Indices assist in classifying components, in finding them in original stars and in the other tables. Indices of two types are used.

Indices of S_i or S^i type point to the star to which the component belongs and to the vertex with which the component is associated. Such indices appear as follows: 1_1 , 2^5 etc. The large digit (S) is the number of a star and the small digit (i) is the number of a vertex. For example, 1_3 or 1^3 mean that the given variation of the given component belongs to the first star and is associated with the third vertex. For sides, triads, equilateral triangles, hexads and dodecads, the vertices of an original star are

read in the following order: in the case of s_i vertices are read from i clockwise; in the case of s^i vertices are read towards i counterclockwise. Therefore, s^i is an inverse variation with respect to s_i , its vertices (notes) are read in opposite direction. Chord s^i is a full inversion of chord s_i . Melody s^i is a retrograde form of s_i . In "melodic" tables (such as "Horizontal Sides...") indices are always accompanied by arrows that show to which direction of melody this or that index corresponds. For complements the indices and the reading order of vertices correlate in a different way. If the vertices of a diamond following one another clockwise designate A, B, C, D, where A is the vertex associated with the given variation, then for variation s_A the order of vertices is $\langle ABDC \rangle$ and for variation s^A the order is $\langle ADBC \rangle$. Angles and crosses do not have their own indices and are described by the indices of the sides of which they are made up. A double or multiple index, such as $5_{16}1$ or $1_{12}2_{14}1$, means that the given component belongs to two or several stars. Parentheses, as in $1_3(4^6)$, mean that the corresponding structure can be obtained by replacing all or some outer vertices by inner ones and vice versa. Within components of a given class there is a one-to-one relation between variations and indices.

Indices of F or F.G type point to the family and genus to which a given component belongs. Such indices appear as 1, 2.2 etc. The first digit (F) is the family. If a family has more than one genus, this is shown by a digit preceded by a dot (.G). Components of a certain family belong to the same genus if they have vertices of the same value (the order of vertices is not important). For example, sides $\langle 3,12,5,6 \rangle = 1_3$, $\langle 6,5,12,3 \rangle = 1^3$ and $\langle 5,12,3,6 \rangle = 4_3$ belong to family/genus $\{3,5,6,12\} = 6.2$. Components of the same family and genus sound like inversions of the same chord. Components of a certain class belong to the same family if the vertex values of one component can be transformed into vertex values of another by adding a constant to each value and subtracting 12 when the sum appears 13 or more. For example, sides $\langle 3,12,5,6 \rangle = 1_3$ and $\langle 2,12,9,3 \rangle = 6_{11}$ belong to the same family (6) since $\text{mod}_{12}^1(\{2,3,9,12\} + 3) = \{(2+3),(3+3),(9+3),(12+3-12)\} = \{5,6,12,3\}$. Components of different genera of the same family sound like different transpositions and possible different inversions of the same chord. A full list of families and genera for magic tetra-, tri-, hexa- and enneachords is found in the tables "All n-Note Combinations..." (p. 18.1-8).

* * *

The stars in their original form are represented in the tables "Vertical Stars..." (p. 1.1-14) and "Horizontal Stars..." (p. 2.1-22) from Volume 1. Notes are distributed over octaves and over a plane of a staff so that the shape of a star may be discerned. Vertices (notes) are connected by lines for the sake of visual clarity. Numerals near notes correspond to the vertex values, for instance, each E is always accompanied by '1'.

The remaining tables from Volume 1 represent stars in the form of different "evolvents" made up of certain components. The components are written as chords or as melodies.

"Stars in Base-n Mode" (p. 11.1-12), which open Volume 2, again show stars in their original form. However, values are now written in different numerical systems (from base-2 to base-13) and the notes correspond not to values but to ciphers ('A', 'B' and 'C' mean '10', '11' and '12', respectively).

In "Ordered Sides...", "Ordered Complements..." etc. the arrangement of the components is based on voice-leading. The priority of voices (the priority may be different in different cases) and their successive ascending or descending motion are determinative. For example, in "Ordered Sides 1" (p. 12.1) the bass, which moves up (reading

left-to-right), is considered first: all chords on *E* are followed by all chords on *F* etc. The tenor, which moves down from *D sharp*, is considered second. Then, when necessary, the alto is considered next. To the left and to the right of the music symbols are placed in columns. The number of symbols equals the number of voices in the chords. The symbols reflect the voice movement and voice priority in relation to each other. 'U' or 'u' means upward motion, 'D' or 'd' means downward motion. The symbols, in descending priority, are as follows:

U or D
u or d
(U) or (D)
(u) or (d)
((u)) or ((d))

For voices with the lowest priority '-' may replace the above-listed symbols. Symbols to the left of the music correspond to left-to-right reading; symbols to the right of the music correspond to right-to-left reading. In melodic tables (such as "Ordered Horizontal Sides...") the symbols are placed not in columns but in lines. The number of symbols in a line corresponds to the number of notes in the melodies.

In "Sides Sorted by Structure", "Complements Sorted by Structure" etc. the components are distributed among sections so that a certain section represents all the chords of a certain family. A *generalized interval structure of a chord* (disregarding the actual order of notes and their octave position) is described as "(0459)", "(01235689A)" etc.: '0' is a certain pitch-class (a note irrespective of its octave position) considered basic, '1' is a pitch-class one semitone higher than the basic, '2' is two semitones higher, etc. ('A', 'B' and 'C' mean '10', '11' and '12' respectively). An extra record like "(0=<pitch-class name>)" is also possible. For example, "(0=e)" means that the basic pitch-class for all chords of this section is *E*.

"All n-Note Combinations..." represent all families and genera for all magic tetra-, tri-, hexa- and enneachords including those that exist neither in stars, nor in the other tables. For combinations absent in the stars, indices F.G look like F.-, F.-G, -F, or -F.G and indices S_i or Sⁱ are replaced by '-'.

"Antistars" are stars in which inner vertices are moved outside and outer vertices are moved inside. In other words they are stars as if turned inside out. Antistars lose the magic properties of the stars and the chords become different.

★ ♦ ♣

While the total number of tables in *Magic Stars* is large enough for the composition formed by them to look finished, we stress the fundamental incompleteness of the given series and its openness to both interpretations and continuations.

★ ♦ ♣

In most tables systems of two linked staves are used; an imaginary central line between them is common to both staves. The *C* clef marks this central line as the *C* of the first octave: notes on the higher staff are actually read in the treble clef and notes on the lower staff in the bass clef. Systems of four linked staves are also used: notes on the uppermost staff are read in the treble clef two octaves higher and notes on the lowermost staff in the bass clef two octaves lower. Sometimes music is written on non-linked staves; each staff of this sort has its own clef.

Outer (odd) vertices of the stars are always written as white notes and inner (even) vertices as black notes. In some tables there are dots near certain notes, namely: dots are near notes that form outer triads having vertex '1' and inner triads having vertex '12'.

A numeral inside a circle in the score is always the number of a star. Rotation (variation) of a star is designated by a Roman numeral (from I to VI). Reflected variations IR-VIR are not written down explicitly, but they may be obtained by playing (reading) variation I-VI right-to-left.



For a performance one can select any tables or sections and arrange them in any order. (Symbol ' \rightleftharpoons ' marks where the possibility of moving from one table or section to another is provided explicitly.) The music can be read left-to-right or right-to-left. Tempo, rhythm, articulation and the other sound characteristics are left to the performer's discretion.

A slash '/' between two chords (as in "Parallel Sides and Complements", p. 5.1-2) means that it is sufficient to play only one of these chords. If there are notes of the same pitch in the adjacent chords, they can be played as if these notes were tied. (In the tables "Triads Sorted by Structure", p. 14.13-16, the ties are explicit, but may be ignored.) The notes in chords may be played nonsimultaneously. In chord sequences one or more voices may be omitted if desired (e.g. a four-voice progression can be played as a three-, two- or one-voice sequence). In melodic tables any "voice" (every nth-note in a melody) can be transposed to another octave or omitted. Some tables ("Horizontal Stars", p. 2.1-6, "Hexads Sorted by Rotations...", p. 7.11-14 и "Ordered ... Triads...", p. 14.1-12) are prefaced by executive formulas in the form of rhythmic or other figures. However these formulas may be ignored if desired.

A chord with '(8)' above or below it can be played an octave higher or lower, respectively. '(8)' relates to all notes of a chord irrespective of whether they are written on one or on several staves. A dashed line after '(8)' extends the effect onto the group of subsequent chords: all these chords should be transposed in the same manner. A clef with '(8)' above or below means that the whole table or its corresponding segment can be played an octave higher or lower. An individual note with a stroke ' \nearrow ' or ' \searrow ' to the right of it can be played an octave higher or lower, respectively. A note with a double stroke can also be transposed two octaves. In a chord the notes with strokes may be transposed independently of each other, but without crossings (i.e. among two notes, the higher one must also remain such after transposition) (Fig. 6). Sometimes the possibility of transposition is expressed by a small note, when this note is *exactly* one octave distant from the regular note: in this case the small note can be played *instead* of the regular one, when desired (or when necessary).

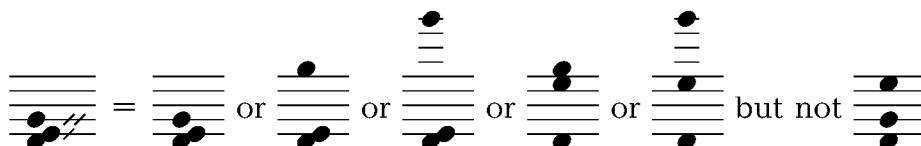


Fig. 6

In other cases small notes may be played *instead of* or *together with* regular notes. In "Ordered Enneads..." (p. 16.1-12) the lower and the higher notes of each chord belong to the same pitch-class (both reflect the same vertex). One of these notes is written as

the regular note (to which the index is related) and the other as a small note. Either of these notes or both can be played. In "Hexads Sorted by Rotations..." (p. 7.11-14) and "... Enneads Sorted by Rotations..." (p. 8.5-12) variants of the chords are written with the small notes; instead of such notation, the sign '/' could be used.

In "... Stars..." one can play the whole stars or choose individual components of them: sides, triads, hexads etc.

In most tables from Volume 1 staff systems, half-systems, double-systems or individual bars are *cycles*, i.e. have a circular structure. The cycle can be started from any place. For example, if the cycle has six components (chords or melodies), besides "1-2-3-4-5-6" or "6-5-4-3-2-1", the progressions "2-3-4-5-6-1", "1-6-5-4-3-2", "3-4-5-6-1-2" etc. are also possible. A performer can also build his own cycles from a given components (for example, components from "... Angles", "... Crosses", "... Enneads..." or "... Dodecads..." can be arranged in a manner similar to that carried out in "... Sides...").

Tables can be played either on the piano or on any other instrument or group of instruments. Any table or group of tables can be played at any interval higher or lower.

* * *

There are also additional possibilities. A performer can arbitrarily change the order of notes or chords within a table or its fragment, make any omissions, repetitions etc. Moreover, not only marked but any notes can be transposed to the other octaves independently of each other, with or without crossings.

However, regarding additional possibilities the following should be kept in mind. The farther the performer departs from the original text, the more probable unsatisfactory or meaningless sounding may become and the less clear may become the very *idea* of magic stars. Nevertheless, in spite of such dangers, I prefer not to set any limitations on a performance (including myself as a performer of this music), as far as possible. Originally magic stars are the world beyond the individual, like the world of nature. As such, it expresses and represents nothing. It does not address us with any emotions or passions. It just exists. Do we have any interest in it and vice versa? However, like many thing in nature, this world can produce a response in us and possibly "say" something important to us. But in order to hear, we are required to come to meet it, to have an open and attentive ear, to be ready to trust our impulses even if they seem "strange". This is a subtle and subjective moment. Limitations coming from *anybody* outside (not directly from nature) may help, but may also become an obstacle for a person who wants to hear and understand something, to know something of *what all this means personally for him*.

July, 2008

Sergei Zagny

ПРЕДИСЛОВИЕ

Настоящая партитура — это серия музыкальных таблиц, в которых представлены магические звёзды и их компоненты в различных формах и сочетаниях. Ноты соответствуют числам, из которых состоят звёзды, комбинации нот — фигурам. Время звучания можно уподобить времени, в течение которого фигуры рассматриваются. В партитуре длительности не определены — время рассматривания может быть разным.

◊ ◊ ◊

Магическая звезда — это шестиконечная звезда, на углах которой (на шести внешних вершинах) и на точках перекрещивания (на шести внутренних вершинах) расположены числа от 1 до 12. У звезды шесть сторон, на каждой — по четыре вершины (две внешних и две внутренних). Числа расположены так, что их сумма одинакова для любой из сторон звезды, а также для шести внешних вершин. Сумма эта всегда равна 26. (Поскольку сумма всех вершин $1 + 2 + \dots + 12 = 78$, всего сторон 6 и каждое число принадлежит двум сторонам, то $78 : 6 \times 2 = 26$.) Всего магических звёзд шесть, если не считать вариаций, образуемых вращениями и отражениями. Каждой звезде присвоен определённый порядковый номер.

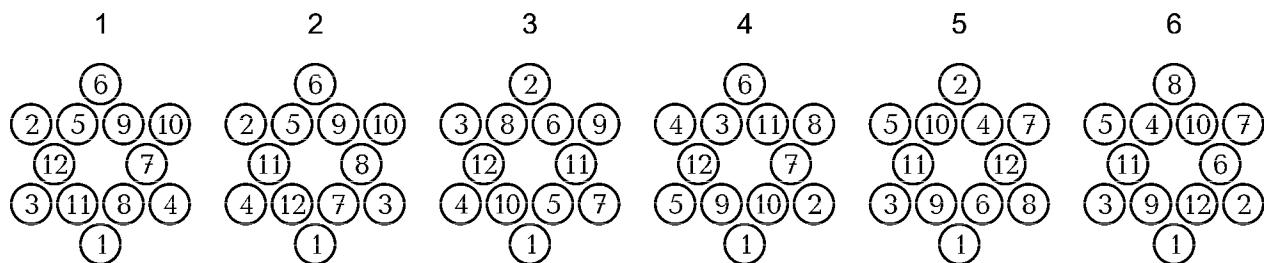


Рис. 1: Магические звёзды, горизонтальная форма

У звезд мы различаем две основные формы: *горизонтальную*, когда две из шести сторон звезды расположены горизонтально (рис. 1), и *вертикальную*, когда две из шести сторон расположены вертикально (рис. 2).

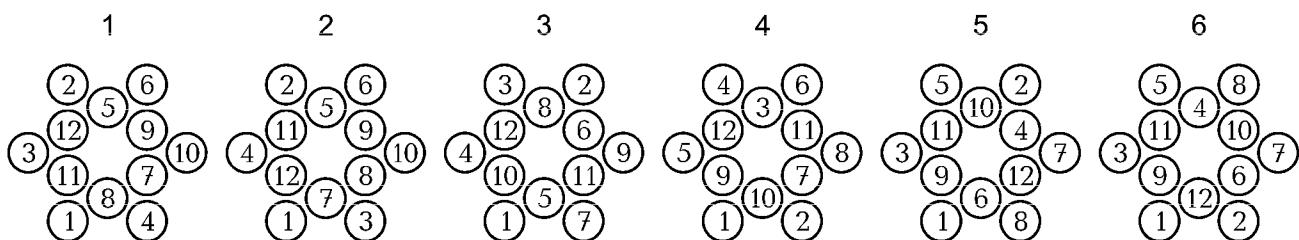


Рис. 2: Магические звёзды, вертикальная форма

Каждая горизонтальная и каждая вертикальная звезда имеет 12 вариаций. Первой вариацией (или исходной формой) горизонтальной звезды (вариацией I или II) считается такая, где «1» находится внизу звезды, а число на верхней левой вершине меньше числа на верхней правой (рис. 1). Поворот вариации I на 1/6 круга против часовой стрелки даёт вариацию II или III, на 2/6 круга — вариацию III или IIII, и т. д. Отражение вариаций I, II, ..., VI относительно вертикальной оси даёт

ещё шесть вариаций — I_R, II_R, ..., VI_R соответственно (рис. 3). Вариации вертикальной звезды имеют те же имена (I, I_R, II, II_R...). Вертикальные вариации I, II, ..., VI получаются путём поворота одноименных горизонтальных вариаций на 1/12 круга по часовой стрелке. Отражение этих вариаций относительно вертикальной оси даёт вариации — I_R, II_R, ..., VI_R.

Вершинам звезды также присвоены порядковые номера. Вершина с числом «1» всегда первая, далее номера последовательно возрастают: для вариаций I-VI по часовой стрелке, для вариаций I_R-VI_R — против часовой стрелки (рис. 3).

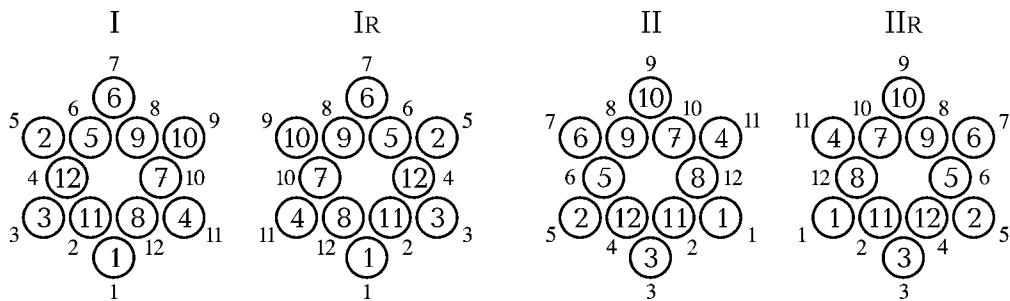


Рис. 3: Горизонтальные вариации I, I_R, II, II_R первой звезды;
числа рядом с ячейками — порядковые номера вершин

Заметим, что номера внешних вершин всегда нечётные, номера внутренних — чётные.

Любую звезду можно представить как $\{n_i\}$, где n — число, а i — номер вершины, где число располагается (рис. 4).

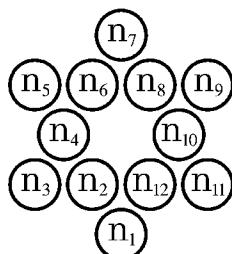


Рис. 4

В звёздах мы различаем следующие компоненты:

стороны (четырёхзвучия) — sides;

дополнения (четырёхзвучия или ромбы) — complements,
нечётные и чётные — odd and even;

триады (трезвучия или треугольники) — triads,
внешние и внутренние — outer and inner;

равносторонние треугольники — equilateral triangles,
внешние и внутренние — outer and inner;

гексады (шестизвучия или шестиугольники) — hexads,
внешние и внутренние — outer and inner;

эннеады (девятизвучия или большие треугольники) — enneads;

додекады (двенадцатизвучия или двенадцатиугольники) — dodecads.

Кроме того, мы различаем следующие сочетания:

параллельные стороны – parallel sides;
противоположные стороны – opposite sides;
углы – angles;
кресты – crosses;
противоположные вершины – opposite vertices.

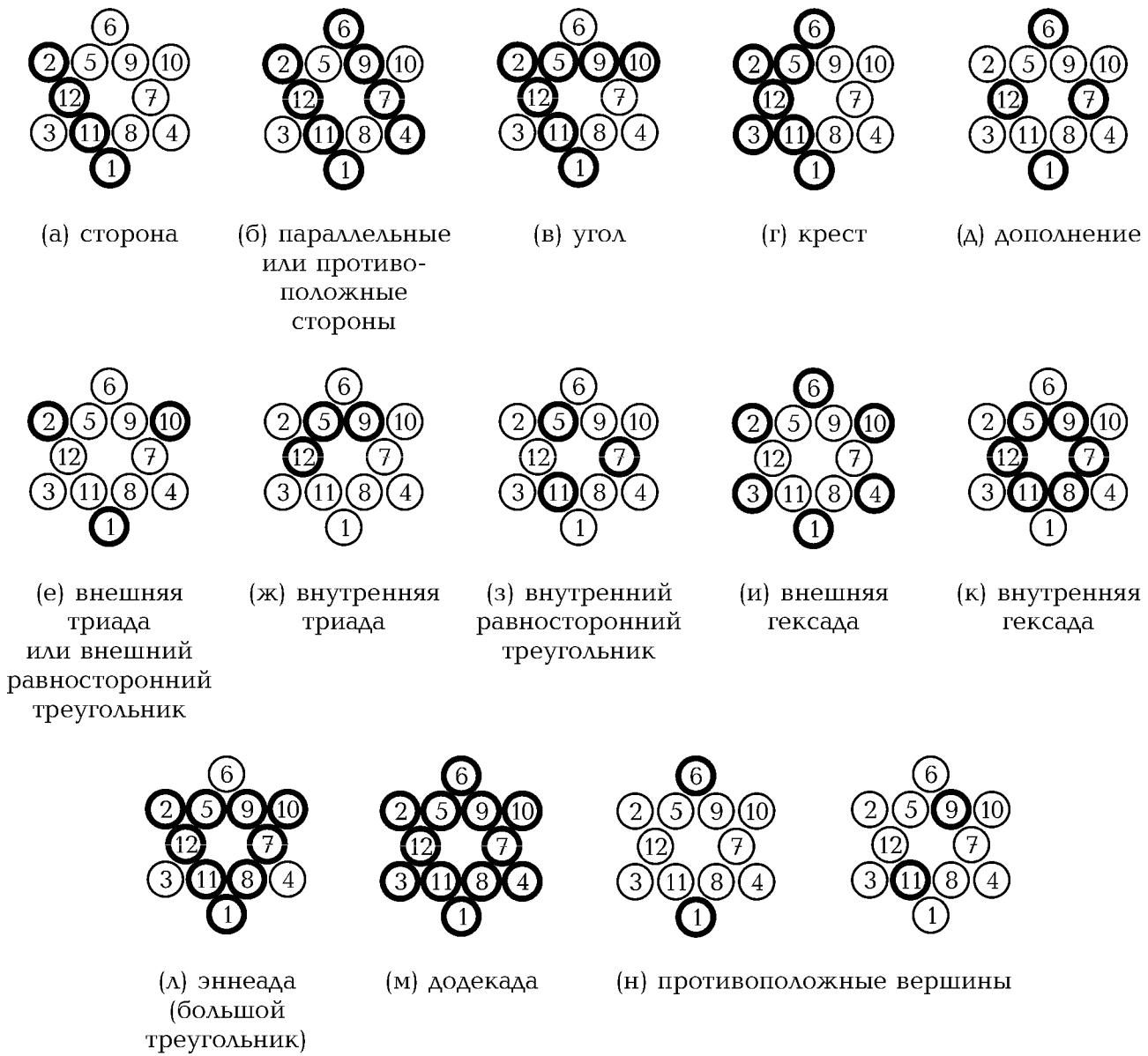


Рис. 5: Компоненты и сочетания

Сторона звезды (она же – сторона одного из двух больших треугольников) – это отрезок, на котором лежат четыре вершины (рис. 5а). На концах отрезка располагаются внешние вершины, внутри отрезка – внутренние. Всего в звезде 6 сторон, сумма вершин любой стороны – 26. Каждая сторона имеет 2 вариации, например, $\langle 1,11,12,2 \rangle$ и $\langle 2,12,11,1 \rangle$. Порядок чтения вершин в вариациях противоположный.

Параллельные стороны – это стороны, не имеющие общих вершин (рис. 5б). Для каждой стороны в пределах звезды существует только одна параллельная сторона.

На параллельных сторонах восемь вершин, их сумма — $26 \times 2 = 52$. Параллельные стороны, если они читаются во взаимно противоположном направлении, называются *противоположными*.

Угол — это две пересекающиеся стороны, имеющие общую внешнюю вершину (рис. 5в). Крест — это две пересекающиеся стороны, имеющие общую внутреннюю вершину (рис. 5г). И угол, и крест состоят из семи вершин, сумма которых непостоянна. Но если точку пересечения считать дважды, то сумма всегда будет $26 \times 2 = 52$.

Если из звезды мысленно удалить любые две параллельные стороны, то останутся четыре вершины, расположенные в виде ромба. Эти вершины называются *дополнением* (рис. 5д). В звезде три дополнения. Сумма вершин каждого дополнения — 26. Вариации дополнений различаются порядком, в каком читаются вершины. В партитуре каждое дополнение представлено восемью вариациями. В четырёх из них первая и последняя вершины — внешние (нечётные), эти вариации называются *нечётными дополнениями*. В четырёх оставшихся вариациях первая и последняя вершины — внутренние (чётные), эти вариации называются *чётными дополнениями*.

Триада — это три внешние вершины, дающие в сумме $26 : 2 = 13$ — *внешняя триада* (рис. 5е), или три внутренние вершины, дающие в сумме 26 — *внутренняя триада* (рис. 5ж). В звезде четыре триады, две внешние и две внутренние. Для каждой звезды они определяются вполне однозначно. В одну из внешних триад всегда входит вершина «1»; в одну из внутренних триад всегда входит вершина «12». Внешняя триада — это три угловые вершины большого равностороннего треугольника (т. е. внешняя триада и *внешний равносторонний треугольник* всегда совпадают, рис. 5е). Напротив, вершины внутренней триады никогда не образуют равносторонний треугольник, а вершины, образующие *внутренний равносторонний треугольник*, в сумме никогда не дают 26 (рис. 5з). Каждая триада и каждый равносторонний треугольник имеют по 6 вариаций, в трёх из них вершины читаются по часовой стрелке, в трёх других — против часовой стрелки.

Гексада — это шесть внешних вершин — *внешняя гексада* (рис. 5и), или шесть внутренних вершин — *внутренняя гексада* (рис. 5к). Сумма вершин внешней гексады — 26, сумма вершин внутренней гексады — 52. В звезде две гексады, одна внешняя и одна внутренняя. Каждая гексада имеет 12 вариаций, в шести из них вершины читаются по часовой стрелке, в шести других — против часовой стрелки.

Эннеада (или *большой треугольник*) — включает в себя три внешние вершины (одну из двух внешних триад) и шесть внутренних вершин (рис. 5л). Сумма вершин эннеады — $13 + 52 = 65$. В звезде две эннеады, в партитуре каждая представлена 12-ю вариациями (первая либо последняя вершина в этих вариациях всегда внешняя). В шести вариациях вершины читаются по часовой стрелке, в шести других — против часовой стрелки.

Додекада — включает в себя все вершины звезды (рис. 5м). Сумма вершин додекады — 78. В звезде одна додекада, она имеет 24 вариации. В двенадцати вариациях вершины читаются по часовой стрелке, в двенадцати других — против часовой стрелки.

Числа, из которых состоят звёзды, преобразованы в ноты следующим образом: числу «1» соответствует нота *ми*, числу «2» — *фа*, и т. д., вверх по хроматической гамме.

В таблицах, где компоненты (стороны, дополнения и т. д.) рассматриваются отдельно, эти компоненты всегда сопровождаются индексами. Индексы помогают классифицировать компоненты, находить их в исходных звёздах и в других таблицах. Используются индексы двух видов.

Индексы вида S_i или S^i указывают на то, в какой звезде компонента находится и с какой вершиной она сопоставлена. Эти индексы выглядят как 1_1 , 2^5 и т. п. Крупная цифра (S) — это номер звезды, мелкая (i) — номер вершины. 1_3 или 1^3 , например, означает, что данная вариация данной компоненты принадлежит первой звезде и сопоставлена с третьей вершиной. Для сторон, триад, равносторонних треугольников, гексад, эннеад и додекад вершины в исходной звезде читаются в следующем порядке: в случае S_i вершины читаются *от* i по часовой стрелке; в случае S^i вершины читаются *по направлению к* i против часовой стрелки. Следовательно, S^i — это вариация, обратная по отношению к S_i , её вершины (ноты) читаются в противоположном направлении. Аккорд S^i — это полное обращение аккорда S_i . Мелодия S^i — это мелодия S_i в возвратном движении. В «мелодических» таблицах (таких как «Horizontal Sides...») индексы всегда сопровождаются стрелками, показывающими, какому направлению мелодии тот или иной индекс соответствует. Для дополнений индексы и порядок чтения вершин соотносятся иначе. Если вершины ромба, идущие по часовой стрелке, обозначить как A, B, C, D, где A — вершина, сопоставленная с данной вариацией, то для вариации S_A порядок вершин — $\langle ABDC \rangle$, а для вариации S^A — $\langle ADBC \rangle$. Углы и кресты собственных индексов не имеют и описываются через индексы тех сторон, которые входят в их состав. Двойной или множественный индекс, например, 5_{16} или $1_12_{14}1$, означает, что данная компонента принадлежит двум или нескольким звёздам. Скобки, например, в $1_3(4^6)$, означают, что соответствующую структуру можно получить, заменив все или некоторые внешние вершины внутренними и наоборот. В пределах компонент данного класса каждая вариация и каждый индекс соотносятся друг с другом взаимнооднозначно.

Индексы вида F или F.G указывают, к какому семейству и роду данная компонента относится. Такие индексы выглядят как 1, 2.2 и т. п. Первое число (F) — семейство. Если семейство имеет более одного рода, то род обозначается числом после точки (.G). Компоненты некоторого семейства принадлежат одному роду, если у них одинаковый состав вершин (порядок вершин значения не имеет). Например, стороны $\langle 3,12,5,6 \rangle = 1_3$, $\langle 6,5,12,3 \rangle = 1^3$ и $\langle 5,12,3,6 \rangle = 4_3$ принадлежат семейству/роду $\{3,5,6,12\} = 6.2$. Компоненты одного семейства и рода звучат как обращения одного и того же звука. Компоненты некоторого класса принадлежат одному семейству, если вершины одной компоненты можно преобразовать в вершины другой, прибавив к каждой вершине одно и то же число и отняв 12, когда результат сложения — 13 или более. Например, стороны $\langle 3,12,5,6 \rangle = 1_3$ и $\langle 2,12,9,3 \rangle = 6_{11}$ принадлежат одному семейству (6), поскольку $\text{mod}_{12}^1 (\{2,3,9,12\} + 3) = \{(2+3),(3+3),(9+3),(12+3-12)\} = \{5,6,12,3\}$. Компоненты разных родов одного семейства звучат как разные транспозиции и, возможно, как разные обращения одного и того же звука. Полный перечень семейств и родов для магических

четырёх-, трёх-, шести- и девятизвучий можно найти в таблицах «All n-Note Combinations...» (с. 18.1-8).

◊ * *

Звёзды в их целостной форме представлены в таблицах «Vertical Stars...» (с. 1.1-14) и «Horizontal Stars...» (с. 2.1-22) из первого тома. Ноты распределены по октавам и по плоскости нотоносца так, чтобы угадывалась фигура звезды. Вершины (ноты) для наглядности соединены линиями. Около нот стоят числа, им соответствующие, например, рядом с каждой *ми* всегда стоит «1».

В остальных таблицах первого тома звезды показаны в виде различных как бы «развёрток», составленных из тех или иных компонент. Компоненты записаны как аккорды или как мелодии.

В «Stars in Base-n Mode» (с. 11.1-12), открывающих второй том, звёзды снова даны в своей целостной форме, но теперь числа записаны в разных системах счисления (от двоичной до тринадцатиричной), а ноты соответствуют не числам, но цифрам («A», «B» и «C» означают соответственно «10», «11» и «12»).

В таблицах «Ordered Sides...», «Ordered Complements...» и др. расположение компонент обусловлено голосоведением. Определяющими являются приоритет голосов (в разных случаях разный) и их последовательное восходящее или нисходящее движение. Например, в «Ordered Sides 1» (с. 12.1) сначала рассматривается бас, движущийся (при чтении слева направо) снизу вверх: сначала идут все аккорды на *ми*, затем на *фа* и т. д. Следующим рассматривается тенор, который движется сверху вниз от *ре* к *гез*. Затем при необходимости рассматривается альт. В таблицах слева и справа от нотного текста стоят знаки, записанные в столбик. Число знаков равно числу голосов в аккордах. Знаки отображают, в каком направлении движутся голоса и каков их приоритет по отношению друг к другу. «U» или «ц» означает, что голос движется в восходящем направлении, «D» или «d» — что голос движется в нисходящем направлении. Знаки, в порядке ослабления приоритета, таковы:

U или D
и или d
(U) или (D)
(u) или (d)
((u)) или ((d))

Для голосов с наиболее низким приоритетом вместо перечисленных знаков стоит «-». Знаки слева от нотного текста соответствуют чтению слева направо, знаки справа от нотного текста соответствуют чтению справа налево. В мелодических таблицах («Ordered Horizontal Sides...» и др.) знаки расположены не в столбик, но в строку. Количество знаков в строке соответствует числу нот в мелодиях.

В «Sides Sorted by Structure», «Complements Sorted by Structure» и др. компоненты распределены по разделам так, чтобы в некотором разделе были представлены все компоненты некоторого семейства. Соответствующая семейству *обобщённая интервальная структура созвучия* (без учёта действительной последовательности нот и их октавного положения) описывается как «(0459)», «(01235689A)» и т. п.: «0» — некоторый звукокласс (нота безотносительно к её октавному положению), принимаемый за точку отсчёта, «1» — звукокласс, расположенный на один полутон выше начального, «2» — на два полутона выше начального, и т. д. («A», «B» и «C» означают соответственно «10», «11» и «12»). Возможна добавочная запись ви-

да «(0=<имя звукокласса>)», например, «(0=e)» означает, что точка отсчёта для всех созвучий данного раздела — звукокласс *ми*.

В «All n-Note Combinations...» представлены все семейства и рода для всех магических четырёх-, трёх-, шести- и девятизвучий, в том числе и таких, которые ни в звёздах, ни в других таблицах не встречаются. Для таких несуществующих в звёздах комбинаций индексы F.G имеют вид F-, F-G, -F, или -F.G, а индексы S_i или Sⁱ заменены на «-».

«Antistars» — это звёзды, у которых внутренние вершины оказались снаружи, а внешние — внутри, т. е. звёзды, как бы вывернутые наизнанку. В антизвёздах магические свойства звёзд теряются и созвучия становятся другими.

★ ◊ ◊

Хотя общее количество таблиц в *Магических звёздах* достаточно велико и образуемая ими композиция может выглядеть завершённой, обратим внимание на принципиальную неполноту данной серии и на её открытость как для толкований, так и для продолжений.

★ ◊ ★

В большинстве таблиц используется система из двух связанных нотоносцев, мысленная срединная линия между которыми — общая для обоих нотоносцев. Ключ «До» отмечает эту срединную линию как *до* первой октавы: ноты на верхнем нотоносце фактически читаются в скрипичном ключе, ноты на нижнем — в басовом. Используется также система из четырёх связанных нотоносцев, где ноты на самом верхнем нотоносце читаются в скрипичном ключе двумя октавами выше, а на самом нижнем нотоносце — в басовом ключе двумя октавами ниже. Иногда музыка записана на несвязанных нотоносцах, на каждом таком нотоносце стоит свой собственный ключ.

Внешние (нечётные) вершины звёзд записаны в виде белых нот, внутренние (чётные) — в виде чёрных. В некоторых таблицах около определённых нот стоят точки, а именно: около нот, образующих внешние триады с вершиной «1» и внутренние триады с вершиной «12».

В партитуре число внутри окружности — это всегда номер звезды. Поворот (вариация) звезды обозначается римской цифрой (от I до VI). Отражённые вариации I_R-VI_R никогда не выписаны явно, но их можно получить, играя (читая) вариации I-VI справа налево.

★ ★ ◊

Для исполнения можно выбирать любые таблицы или разделы внутри таблиц и располагать их в любой последовательности. (Места, где возможность перехода от таблицы к таблице или от раздела к разделу предусмотрена явно, отмечены знаком «(↔)».) Текст можно читать слева направо или справа налево. Темп, ритм, артикуляция и другие характеристики звучания оставлены на усмотрение исполнителя.

Наклонная черта «/» между двумя аккордами (как, например, в «Parallel Sides and Complements», с. 5.1-2) означает, что достаточно исполнить только один из этих аккордов. Если в соседних аккордах есть ноты одинаковой высоты, их можно ис-

полнять, как если бы они были связаны лигой. (В таблицах «Triads Sorted by Structure...», с. 14.13-16, лиги стоят явно, но исполнять их не обязательно.) Ноты в аккордах можно играть неодновременно. В аккордовых последовательностях при желании можно пропускать один или несколько голосов (например, четырёхголосную последовательность можно играть как трёх-, двух- или одноголосную). В мелодических таблицах любой «голос» (каждую n -ную ноту в мелодиях) можно транспонировать в другую октаву либо пропускать. Некоторым таблицам («Horizontal Stars», с. 2.1-6, «Hexads Sorted by Rotations...», с. 7.11-14 и «Ordered ... Triads...», с. 14.1-12) предпосланы исполнительские формулы — в виде предваряющих основной текст ритмических или иных фигур, но этим формулам при желании можно не следовать.

Аккорд со знаком «(8)» над или под ним можно играть соответственно октавой выше или октавой ниже написанного. Знак «(8)» относится ко всем звукам аккорда, независимо от того, на одном или на нескольких нотоносцах он записан. Пунктирная линия после «(8)» распространяет действие знака на группу последующих аккордов — транспонировать все эти аккорды нужно одинаковым образом. Ключ с «(8)» сверху или снизу означает, что октавой выше или ниже можно играть всю таблицу или соответствующий её раздел. На октаву вверх или вниз можно транспонировать отдельную ноту, если справа от неё стоит штрих «//» или «\» соответственно. Ноту с двойным штрихом можно транспонировать также на две октавы. В аккорде ноты со штрихами можно транспонировать независимо друг от друга, но так, чтобы не возникало перекрещиваний (т. е. чтобы из двух нот более высокая оставалась бы таковой и после транспозиции) (рис. 6). Иногда возможность транспозиции обозначается при помощи мелкой ноты — когда в аккорде она отстоит от обычной *точно* на одну октаву: в этом случае *вместо* обычной ноты при желании (или при необходимости) можно играть мелкую.



Рис. 6

В других случаях мелкие ноты можно играть *вместо* обычных или *наряду* с ними. В «Ordered Enneads...» (с. 16.1-12) нижняя и верхняя ноты каждого аккорда однотипны (ими отображается одна и та же вершина). Одна из них записана как обычная нота (с ней сопоставлен индекс), другая — как мелкая. Играть можно одну из этих нот либо обе сразу. В «Hexads Sorted by Rotations...» (с. 7.11-14) и в «... Enneads Sorted by Rotations...» (с. 8.5-12) мелкими нотами записаны варианты аккордов — вместо такой записи здесь можно было бы использовать знак «/».

Звёзды в «... Stars...» можно играть целиком либо выбирать в них отдельные компоненты — стороны, триады, гексады и т. д.

В большинстве таблиц первого тома строки, полустроки, двоестрочия или отдельные такты являются *циклами*, т. е. имеют круговую структуру. Начинать цикл можно от любого места. Например, если в цикле шесть компонент (аккордов или мелодий), то помимо последовательностей «1-2-3-4-5-6» или «6-5-4-3-2-1» возможны также «2-3-4-5-6-1», «1-6-5-4-3-2», «3-4-5-6-1-2» и т. д. Из заданных компонент исполнитель может также создавать свои собственные циклы (например, компо-

ненты из «... Angles», «... Crosses», «... Enneads...» или «... Dodecad...» можно располагать подобно тому, как это делается в «... Sides...»).

Таблицы можно исполнять на фортепиано или на другом инструменте или группе инструментов. Любую таблицу можно исполнять на любой интервал выше или ниже написанного.

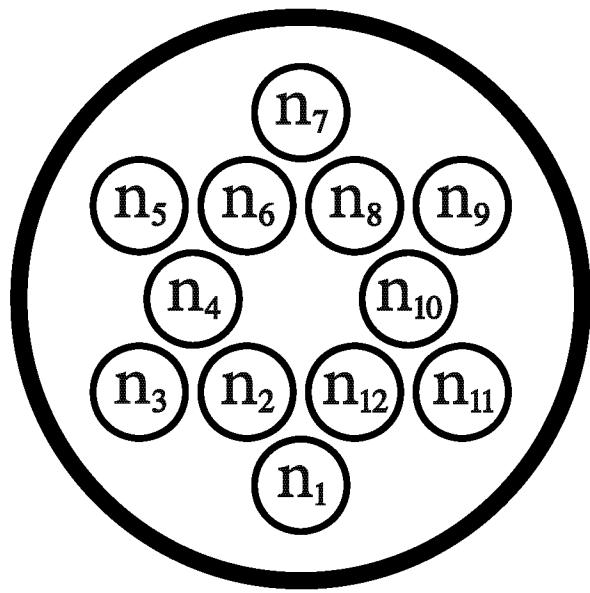
* * *

Существуют также дополнительные возможности. Исполнитель волен произвольно менять порядок нот или аккордов в пределах таблицы или её фрагмента, делать любые пропуски, повторы и т. д. Кроме того, не только отмеченные, но любые ноты независимо друг от друга можно транспонировать в другие октавы, в том числе и с перекрещиваниями.

Относительно дополнительных возможностей, однако, нужно иметь в виду следующее. Чем больше исполнитель удаляется от исходного текста, тем более вероятными становятся неудовлетворительные или неосмыслимые звучания и тем менее различимой может становиться сама идея магических звёзд. Однако, несмотря на такие опасности, я всё же предпочитаю, насколько это вообще возможно, не устанавливать никаких ограничений, связанных с исполнением (в том числе и для себя самого как для исполнителя этой музыки). Изначально магические звёзды — это мир вне субъекта, мир, подобный миру природы. В качестве такового он ничего не выражает и не изображает. Он не адресует нам никаких эмоций и страстей. Он просто есть. Есть ли ему до нас или нам до него дело? Однако, как и многое в природе, этот мир способен вызывать в нас отклик и, возможно, «сообщать» нам нечто для нас важное. Но чтобы услышать, от нас требуется встречное движение, слух открытый и внимательный, готовность доверять своим импульсам, даже когда они кажутся «странными». Это тонкий и субъективный момент. Ограничения, исходящие от кого-либо извне (не от самой природы), могут помочь, но могут и стать препятствием для человека, который хочет что-то услышать и понять, что-то узнать о том, что всё это значит именно для него.

Июль, 2008

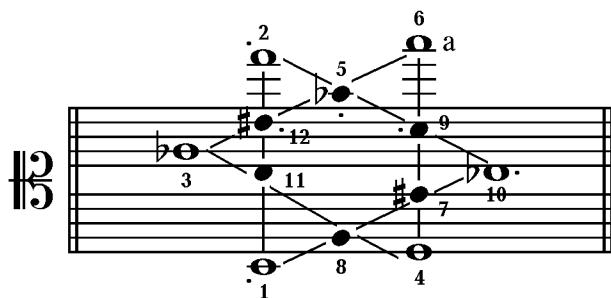
Сергей Загний



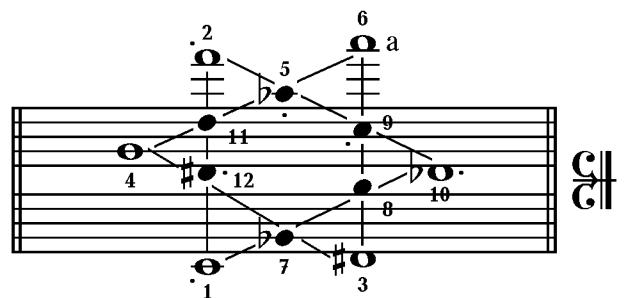
Vertical Stars...

I

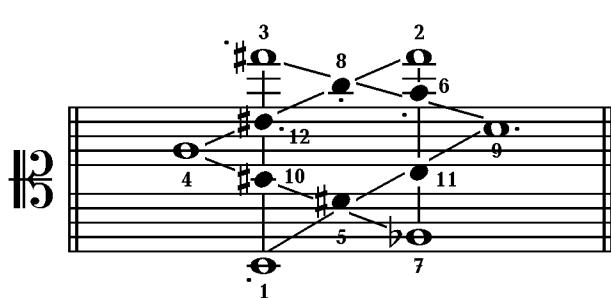
(1)



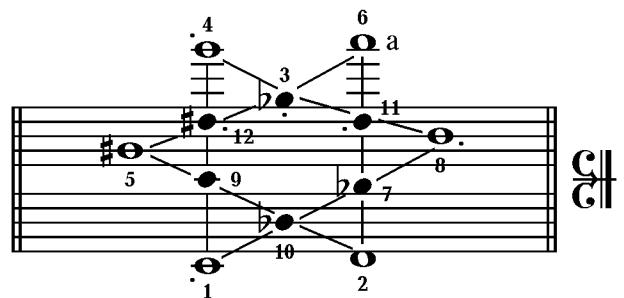
(2)



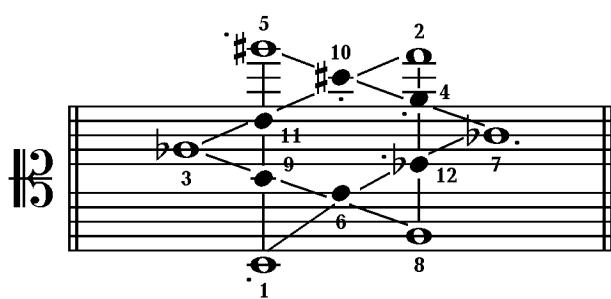
(3)



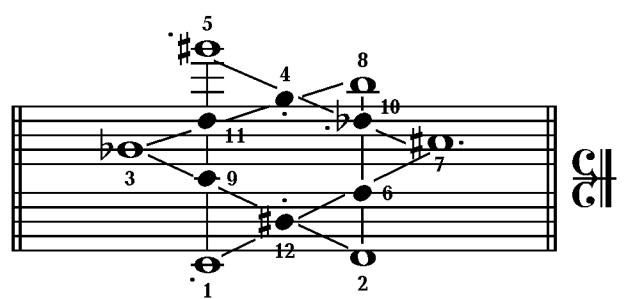
(4)



(5)



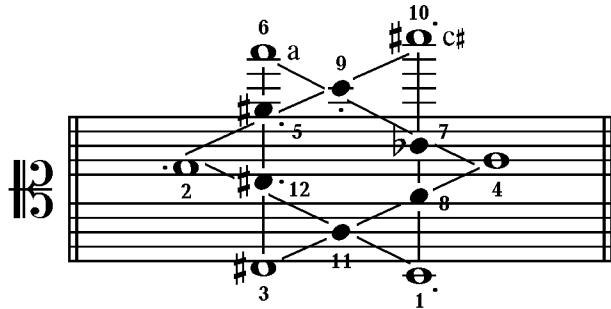
(6)



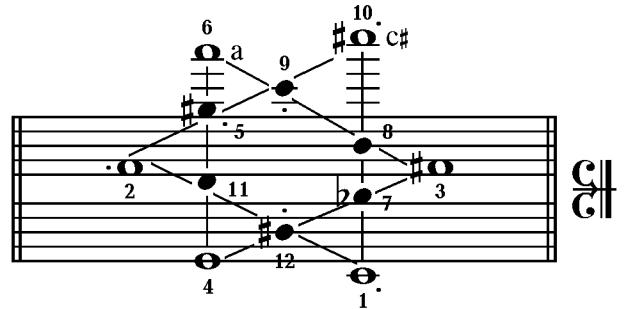
...Vertical Stars...

II

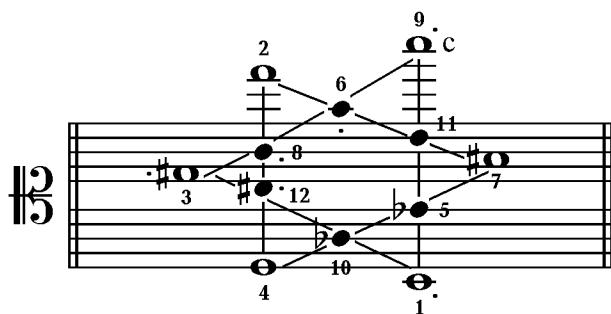
(1)



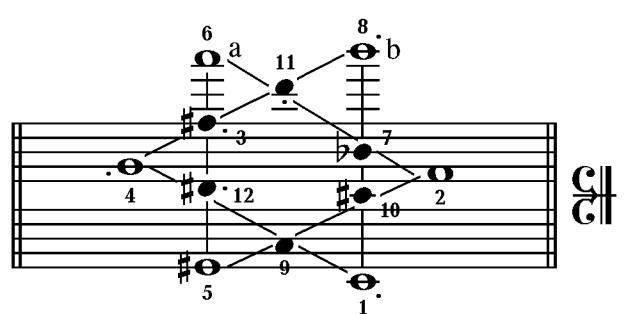
(2)



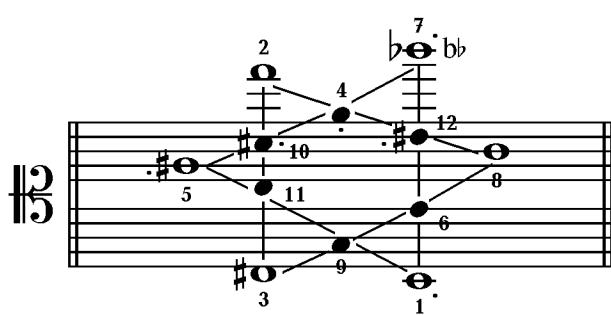
(3)



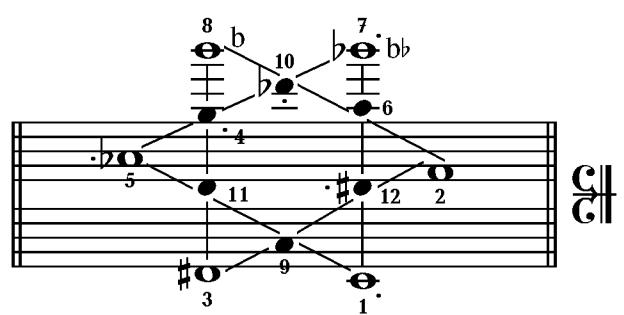
(4)



(5)



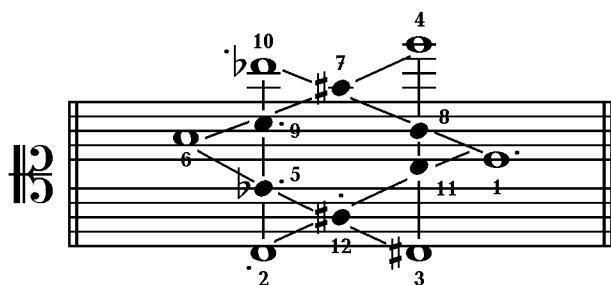
(6)



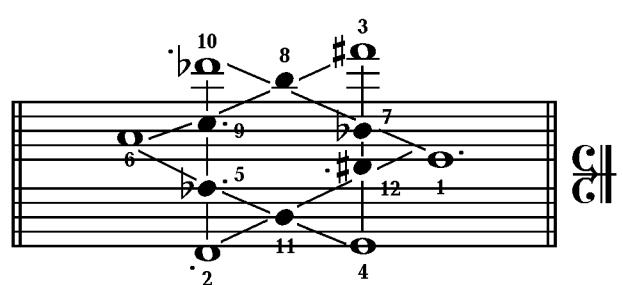
...Vertical Stars...

III

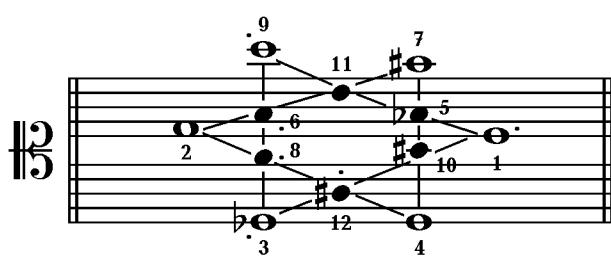
(1)



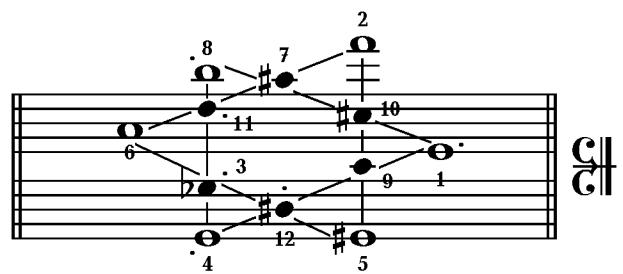
(2)



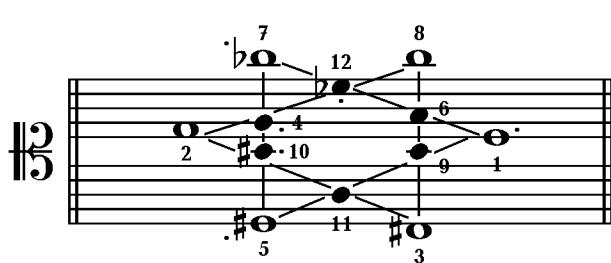
(3)



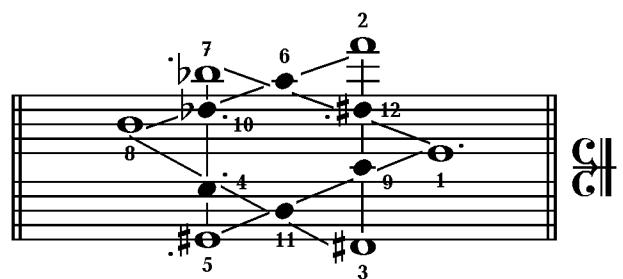
(4)



(5)

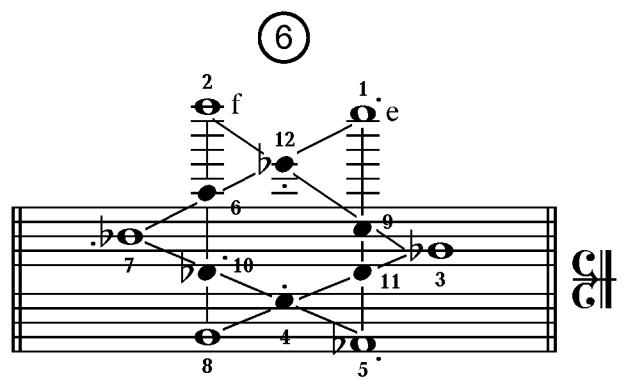
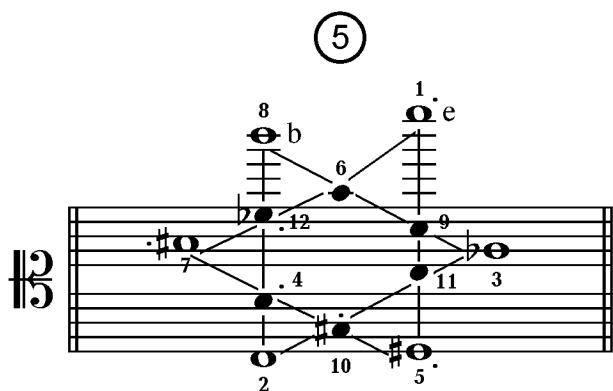
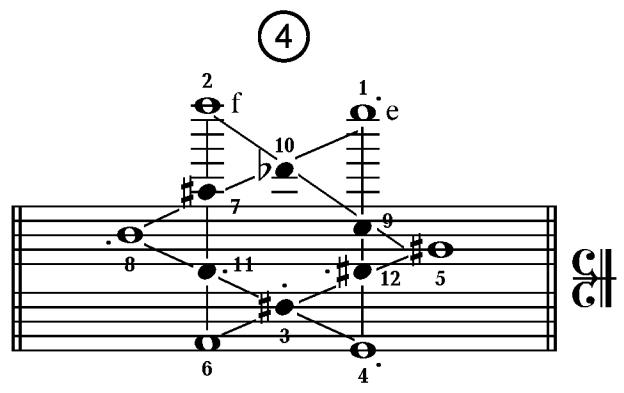
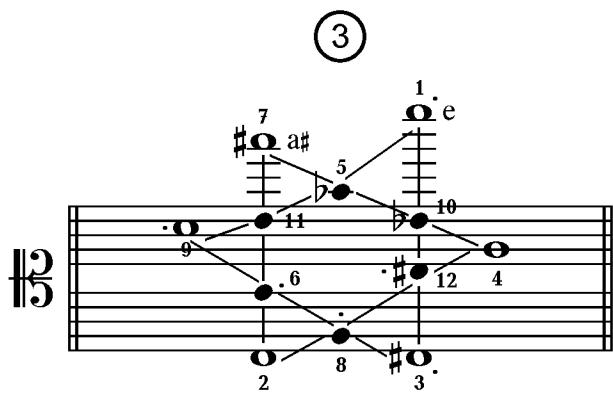
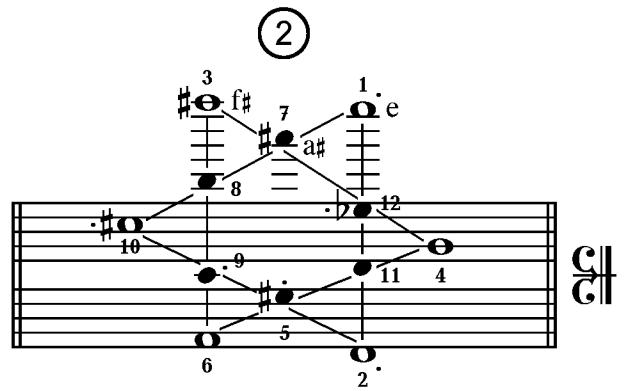
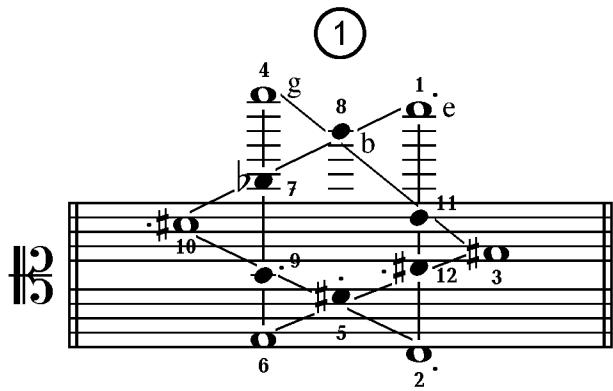


(6)



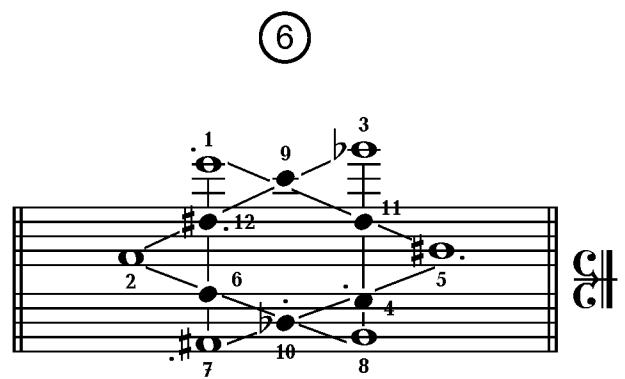
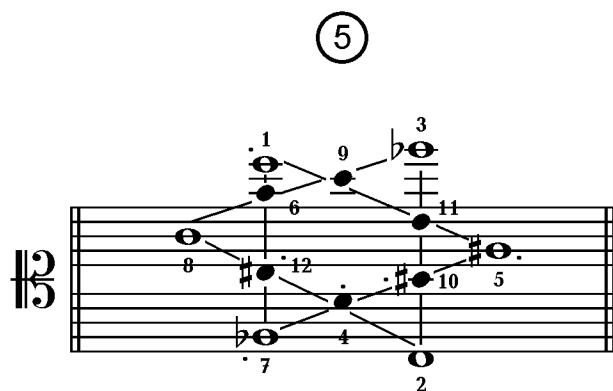
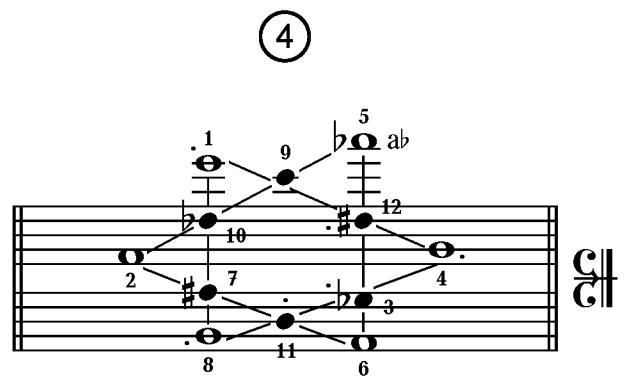
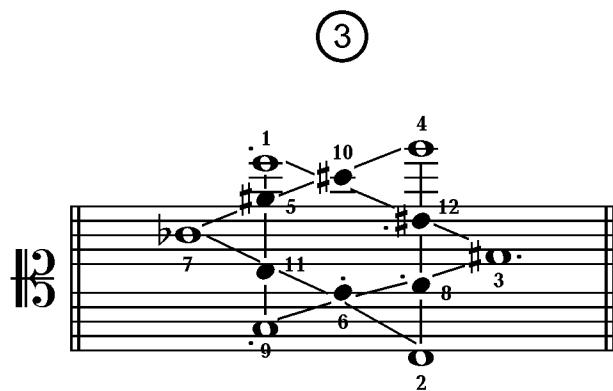
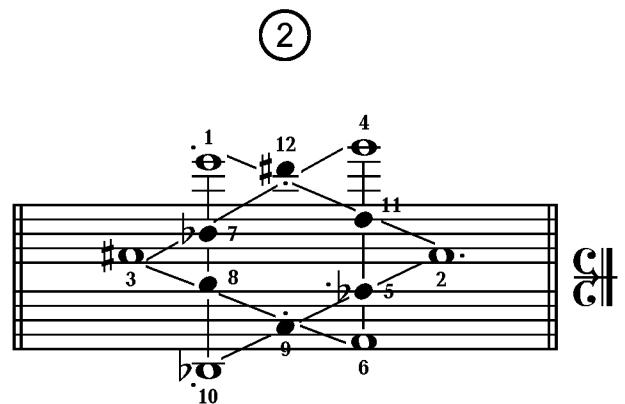
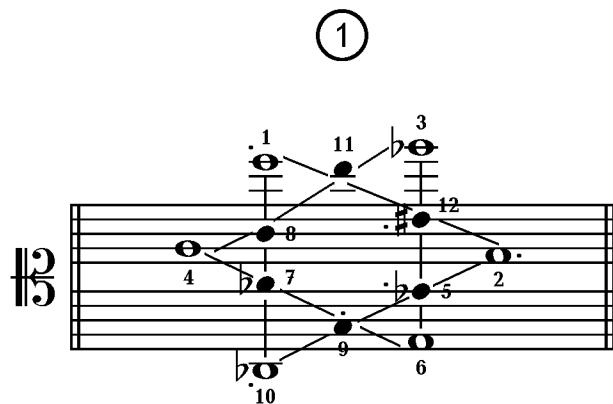
...Vertical Stars...

IV



...Vertical Stars...

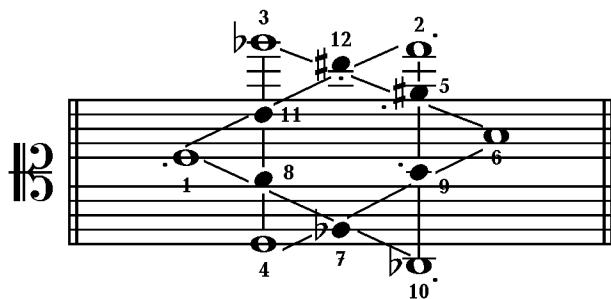
V



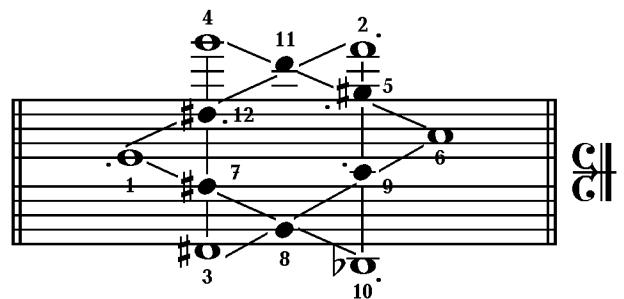
...Vertical Stars

VI

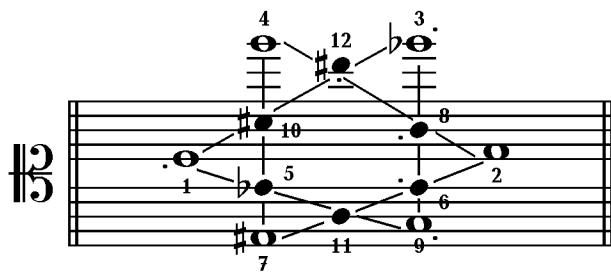
(1)



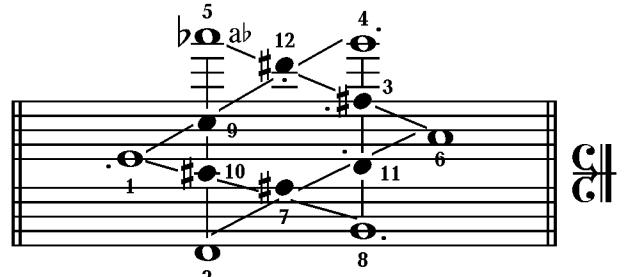
(2)



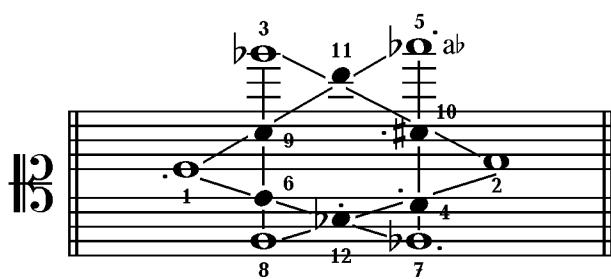
(3)



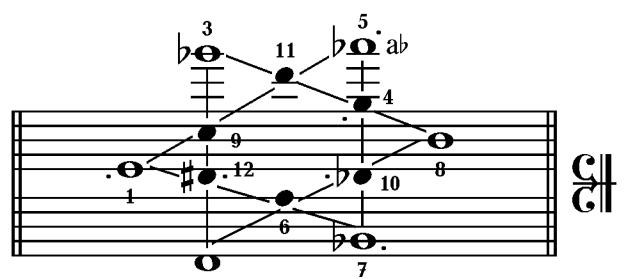
(4)



(5)



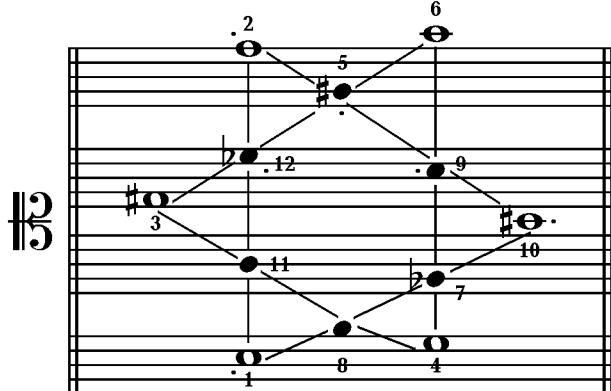
(6)



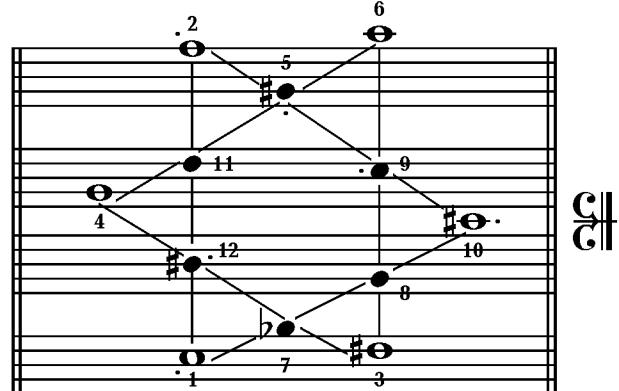
Vertical Stars Expanded...

I

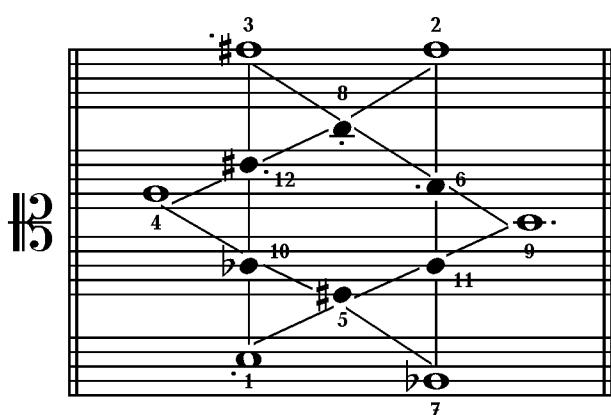
(1)



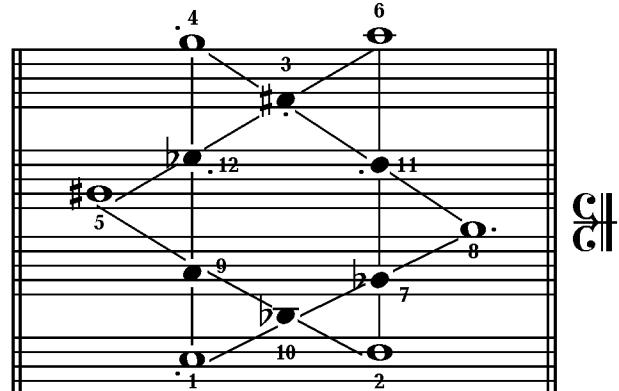
(2)



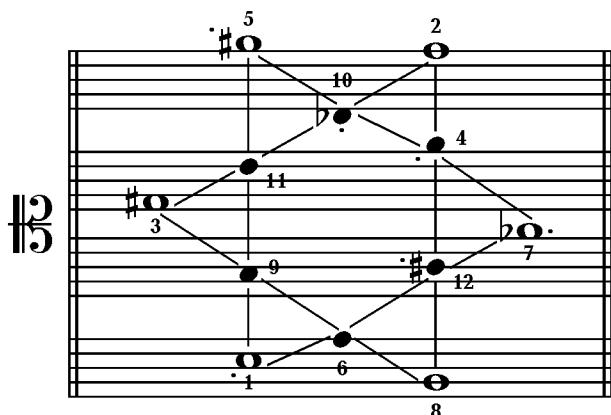
(3)



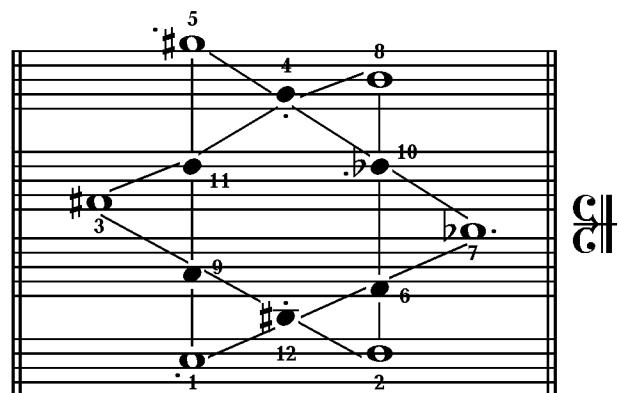
(4)



(5)



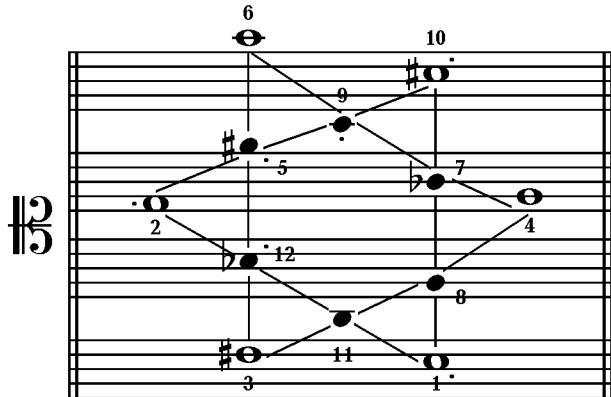
(6)



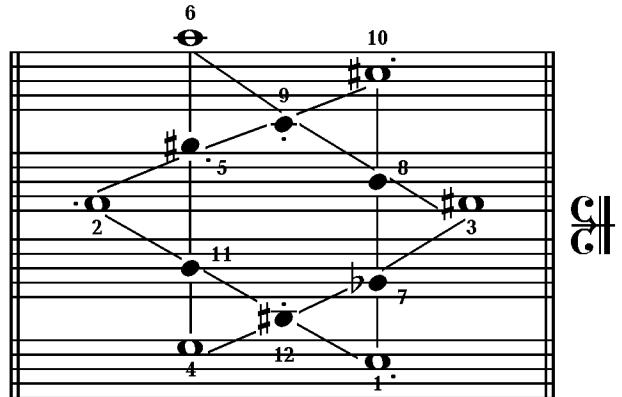
...Vertical Stars Expanded...

II

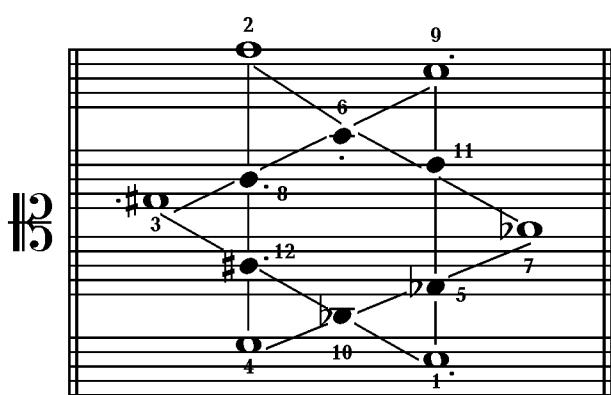
(1)



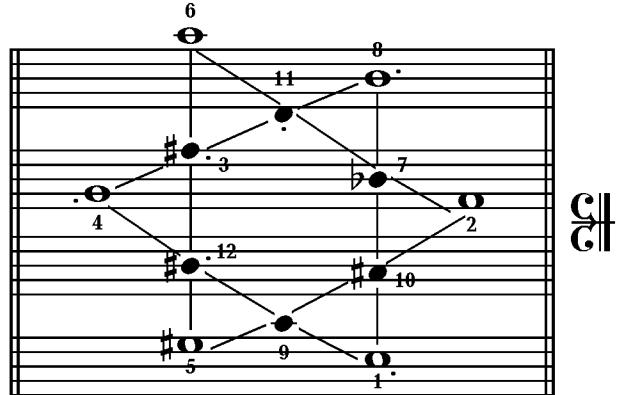
(2)



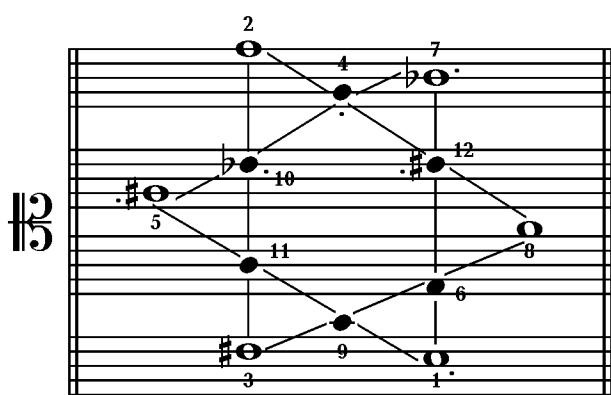
(3)



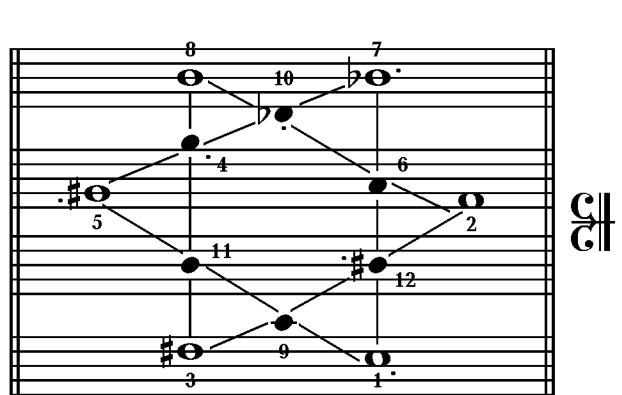
(4)



(5)



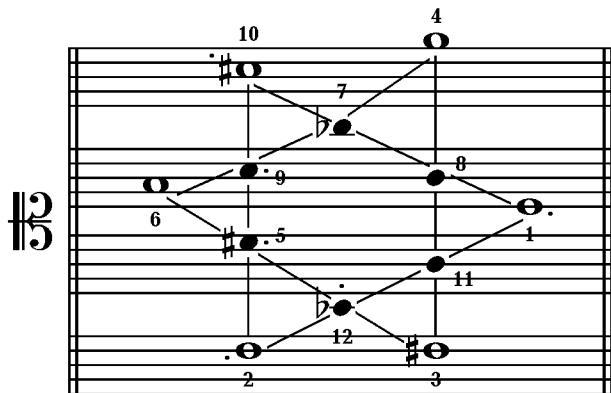
(6)



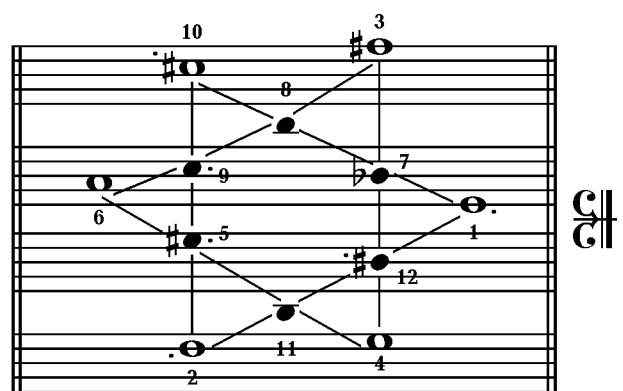
...Vertical Stars Expanded...

III

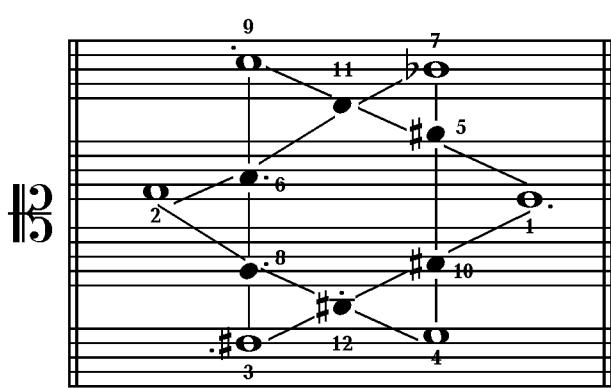
(1)



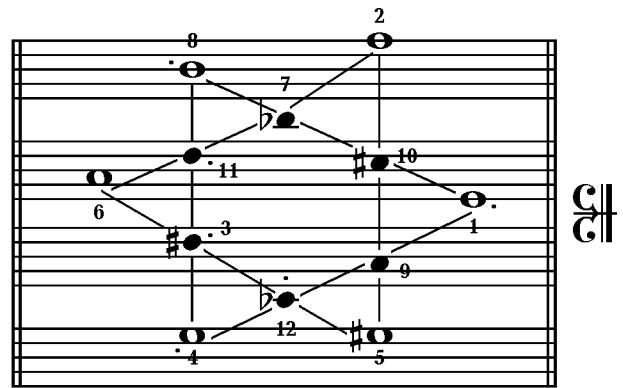
(2)



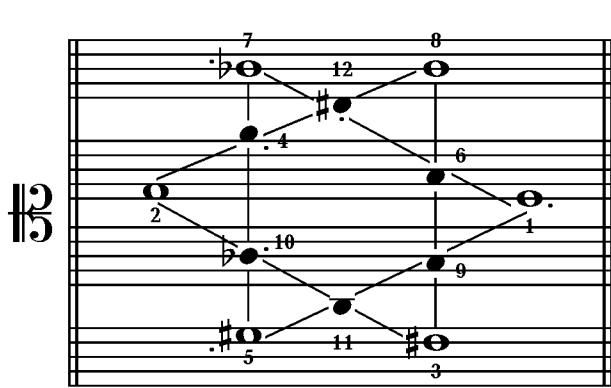
(3)



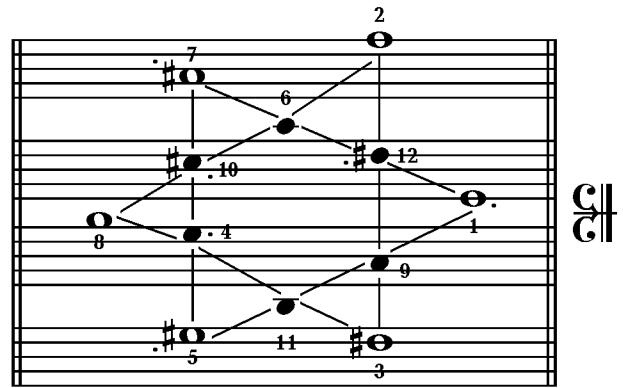
(4)



(5)



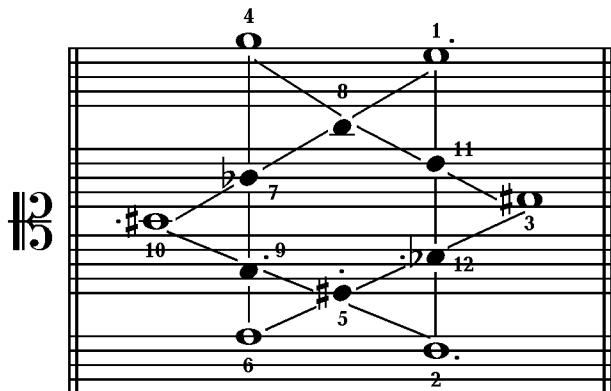
(6)



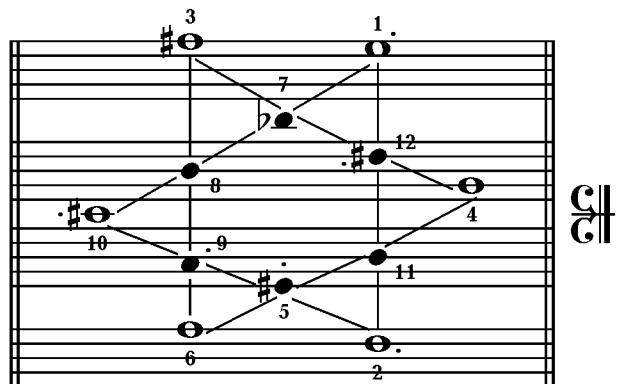
...Vertical Stars Expanded...

IV

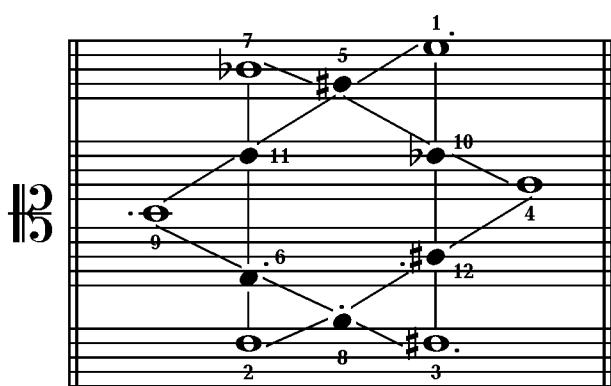
(1)



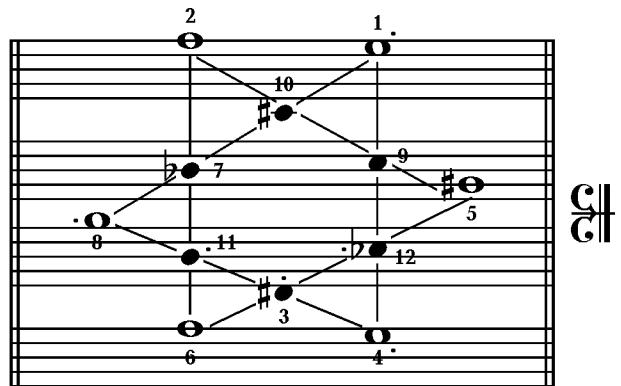
(2)



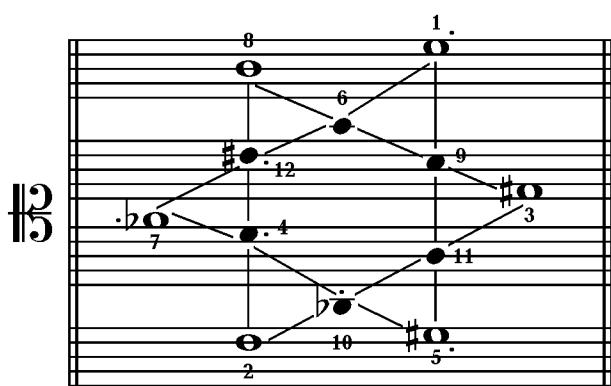
(3)



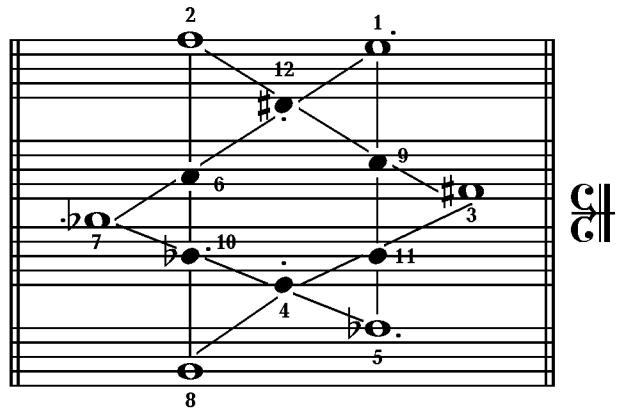
(4)



(5)



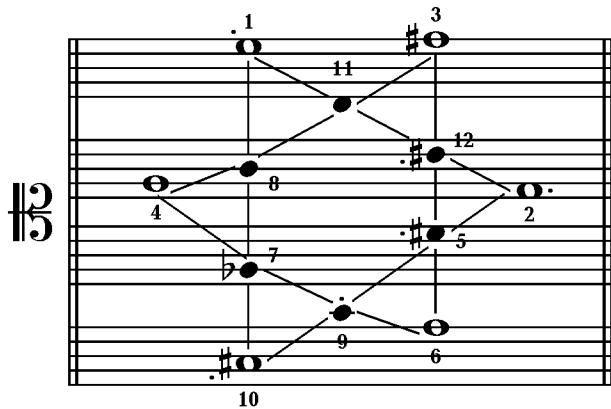
(6)



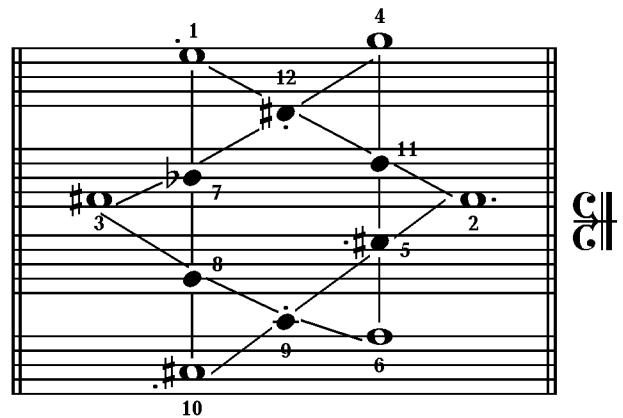
...Vertical Stars Expanded...

V

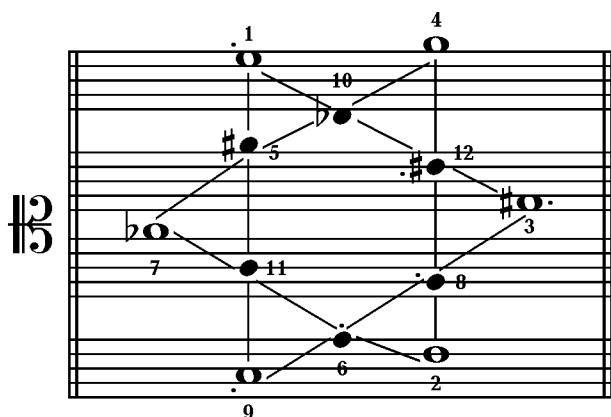
(1)



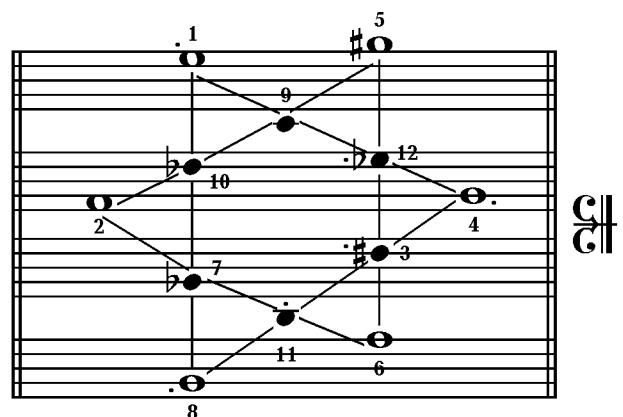
(2)



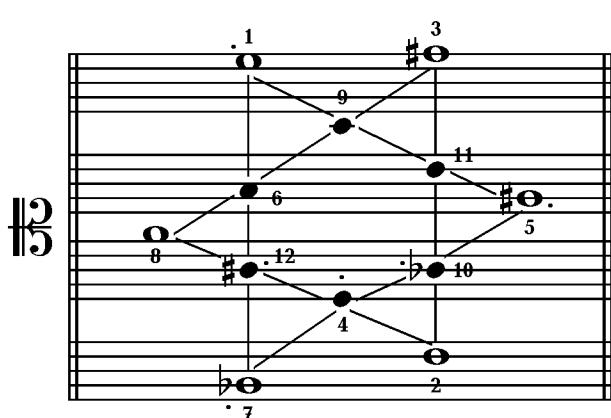
(3)



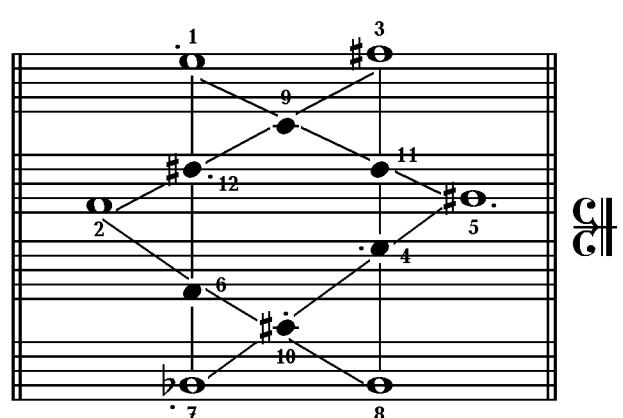
(4)



(5)



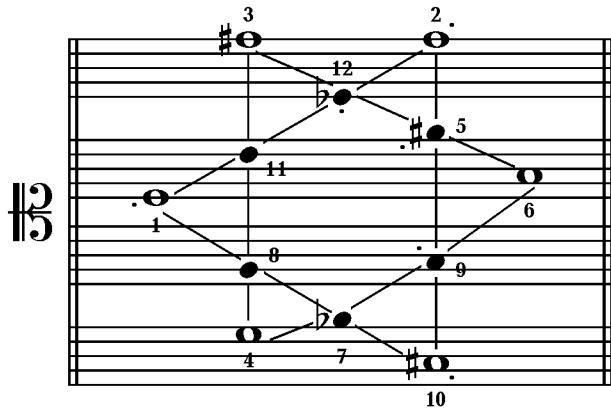
(6)



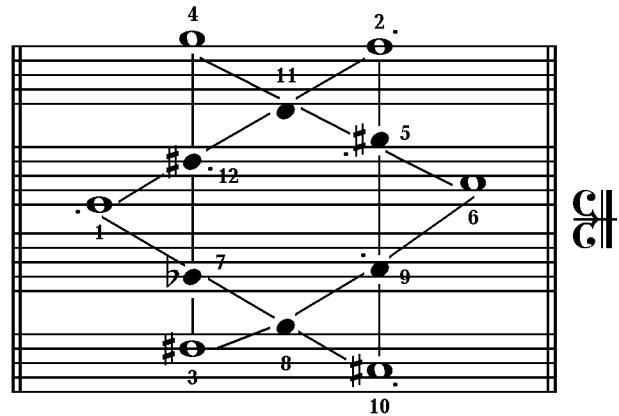
...Vertical Stars Expanded

VI

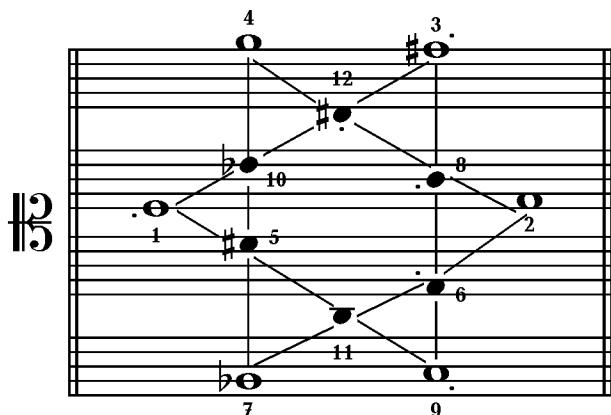
(1)



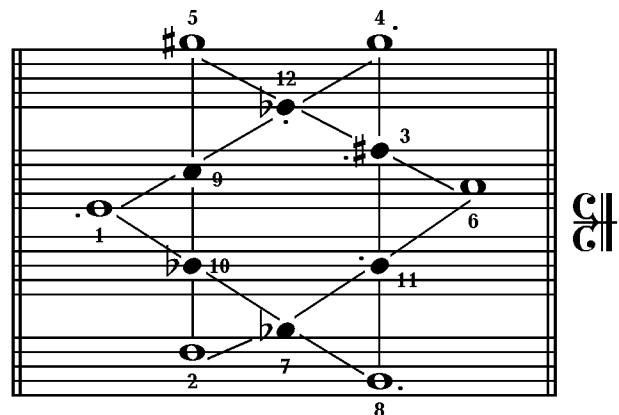
(2)



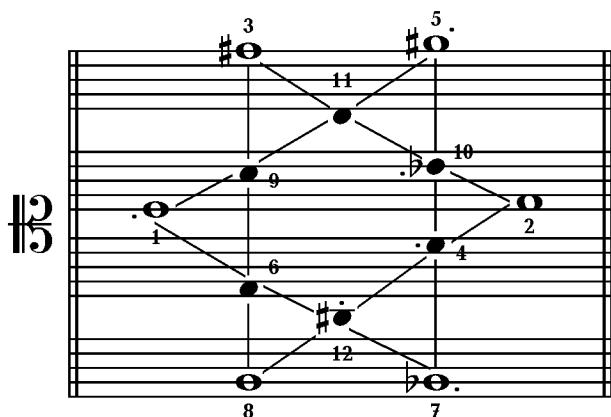
(3)



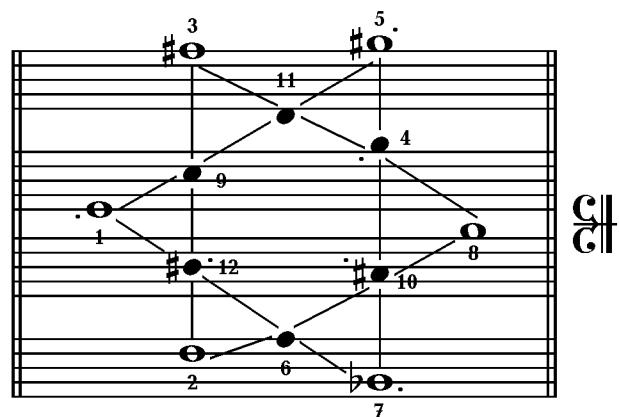
(4)



(5)



(6)



Vertical Stars Compressed...

I

(4)

(5)

(6)

II

(4)

(5)

(6)

III

(4)

(5)

(6)

...Vertical Stars Compressed

IV

Figure showing six staves of compressed vertical star notation for the IV section. Each staff consists of five horizontal lines. Notes are represented by solid black dots. Dashed lines connect notes between staves. Circled numbers 1 through 6 indicate specific patterns or measures.

- Staff 1:** Circled 1. Notes: 4, 7, 8, 1, 11.
- Staff 2:** Circled 2. Notes: 3, 8, 7, 1, 12.
- Staff 3:** Circled 3. Notes: 7, 11, 5, 1, 10.
- Staff 4:** Circled 4. Notes: 2, 7, 10, 1, 9.
- Staff 5:** Circled 5. Notes: 8, 12, 6, 1, 9.
- Staff 6:** Circled 6. Notes: 2, 6, 12, 1, 9.

V

Figure showing six staves of compressed vertical star notation for the V section. Each staff consists of five horizontal lines. Notes are represented by solid black dots. Dashed lines connect notes between staves. Circled numbers 1 through 6 indicate specific patterns or measures.

- Staff 1:** Circled 1. Notes: 1, 8, 11, 3.
- Staff 2:** Circled 2. Notes: 1, 7, 12, 4.
- Staff 3:** Circled 3. Notes: 1, 5, 10, 4.
- Staff 4:** Circled 4. Notes: 1, 9, 5, 12.
- Staff 5:** Circled 5. Notes: 1, 6, 3, 11.
- Staff 6:** Circled 6. Notes: 1, 12, 9, 3.

VI

Figure showing six staves of compressed vertical star notation for the VI section. Each staff consists of five horizontal lines. Notes are represented by solid black dots. Dashed lines connect notes between staves. Circled numbers 1 through 6 indicate specific patterns or measures.

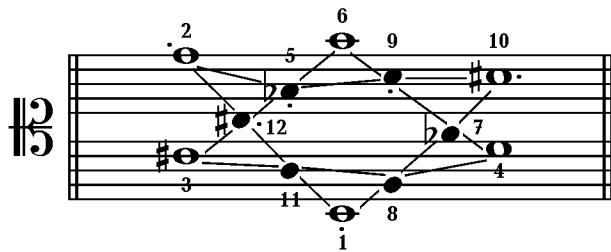
- Staff 1:** Circled 1. Notes: 3, 12, 2, 5.
- Staff 2:** Circled 2. Notes: 4, 11, 2, 5.
- Staff 3:** Circled 3. Notes: 4, 10, 12, 3, 8.
- Staff 4:** Circled 4. Notes: 5, 9, 12, 4, 3.
- Staff 5:** Circled 5. Notes: 3, 9, 11, 5, 10.
- Staff 6:** Circled 6. Notes: 3, 9, 11, 5.

Horizontal Stars...

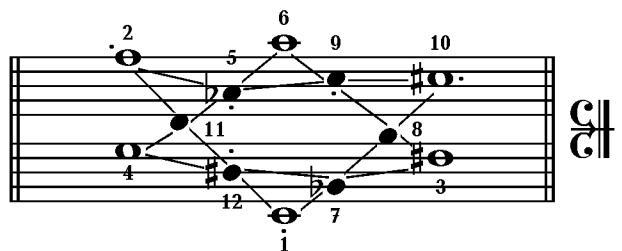
I



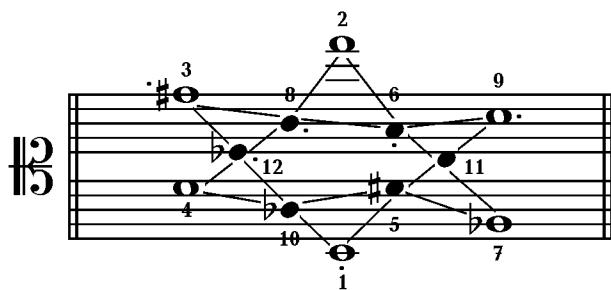
(1)



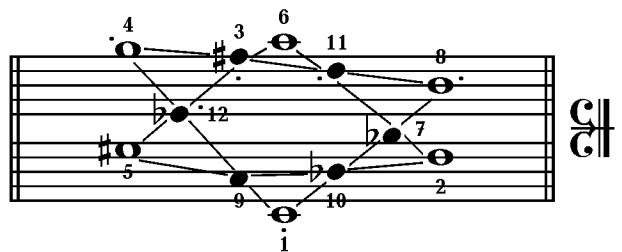
(2)



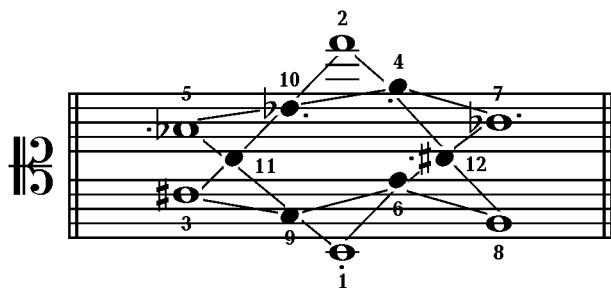
(3)



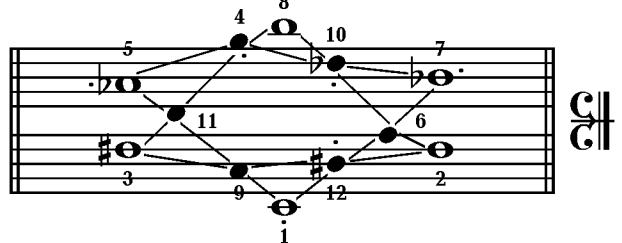
(4)



(5)



(6)

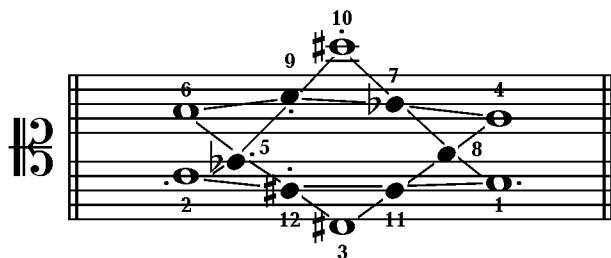


...Horizontal Stars...

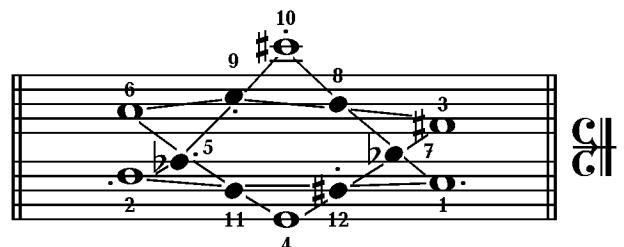
II



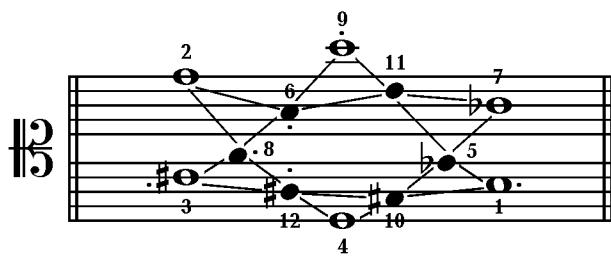
(1)



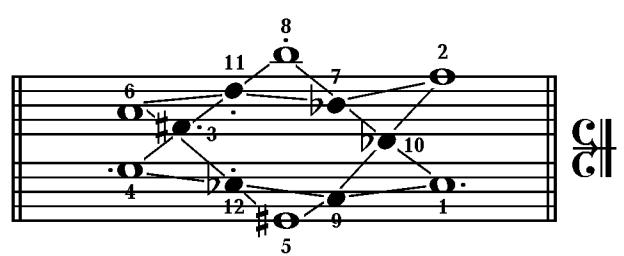
(2)



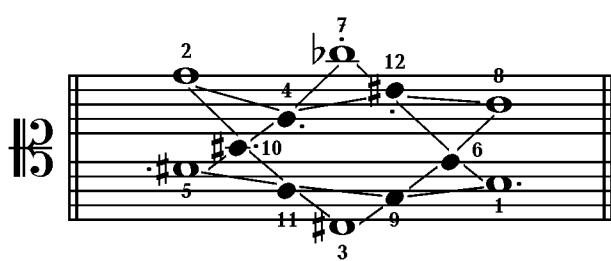
(3)



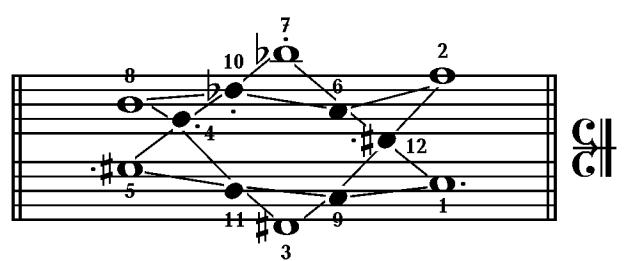
(4)



(5)



(6)

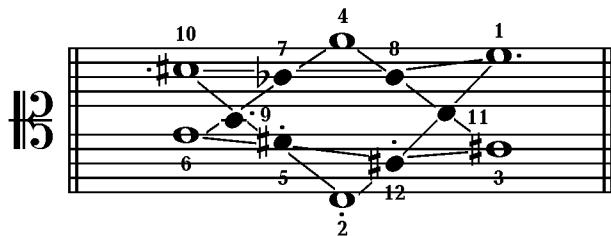


...Horizontal Stars...

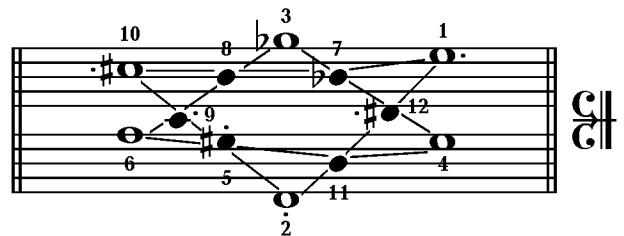
III



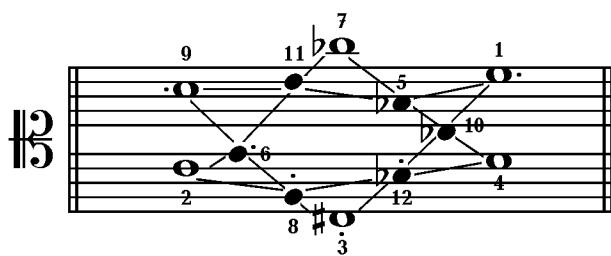
(1)



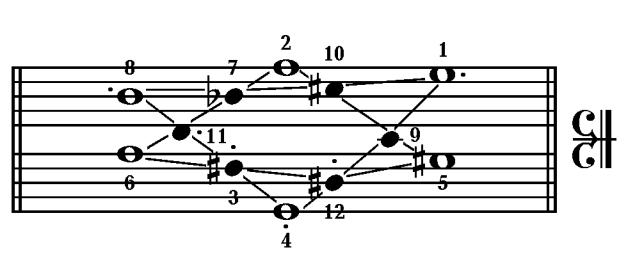
(2)



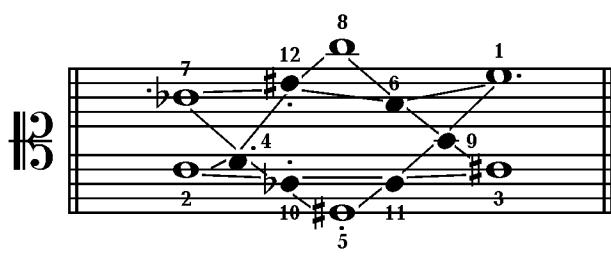
(3)



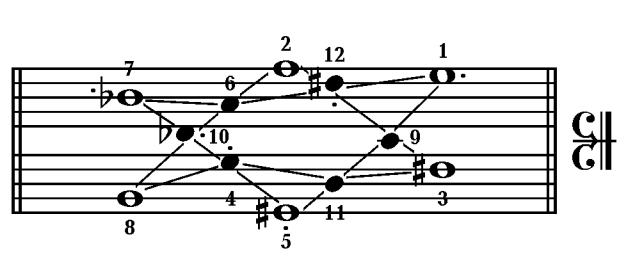
(4)



(5)



(6)

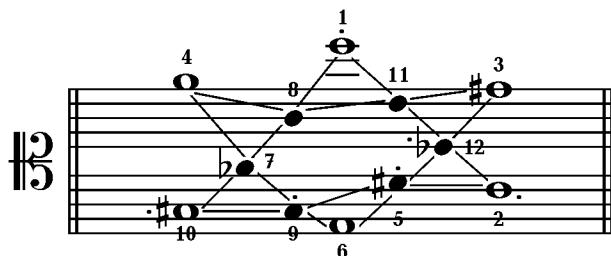


...Horizontal Stars...

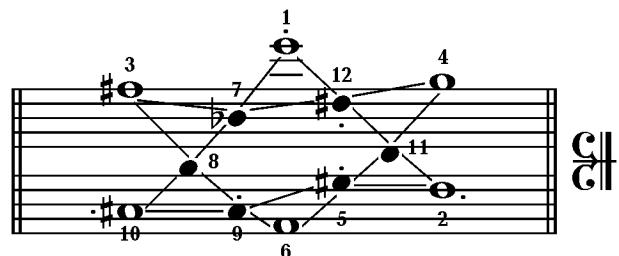
IV



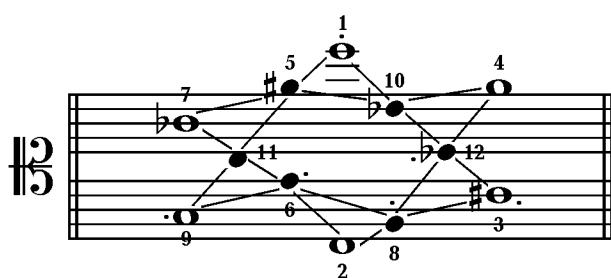
(1)



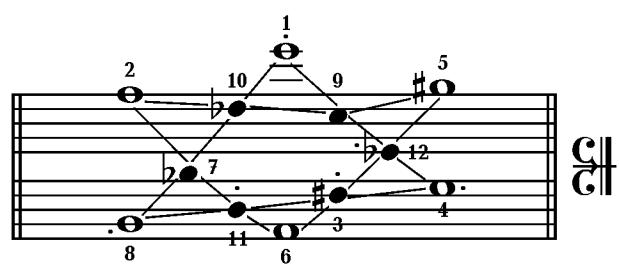
(2)



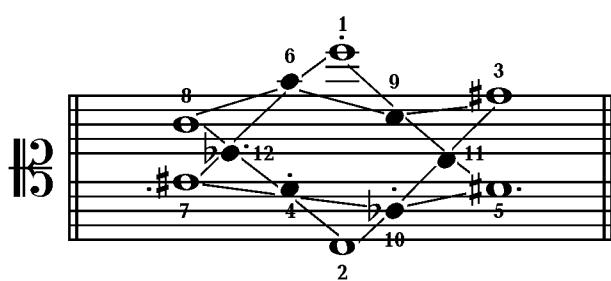
(3)



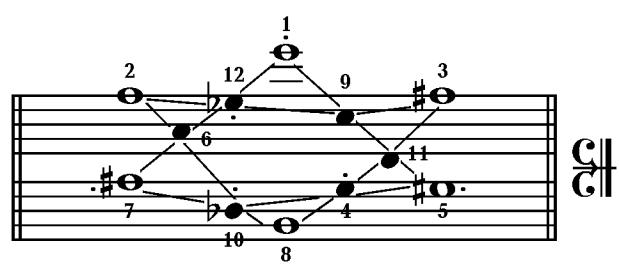
(4)



(5)



(6)

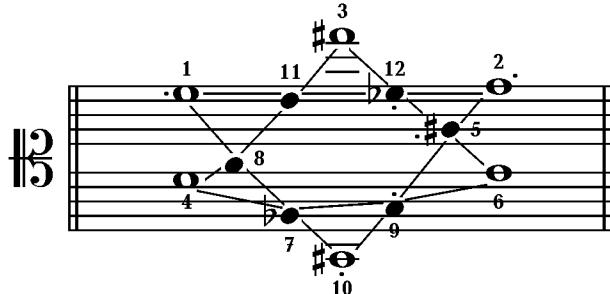


...Horizontal Stars...

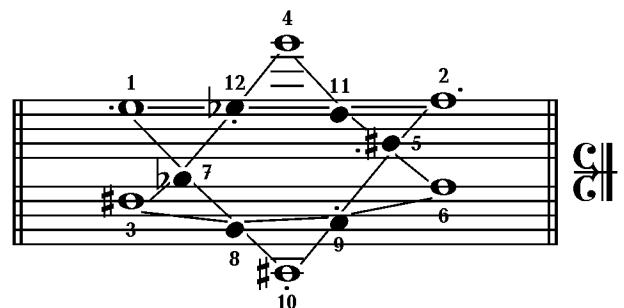
V



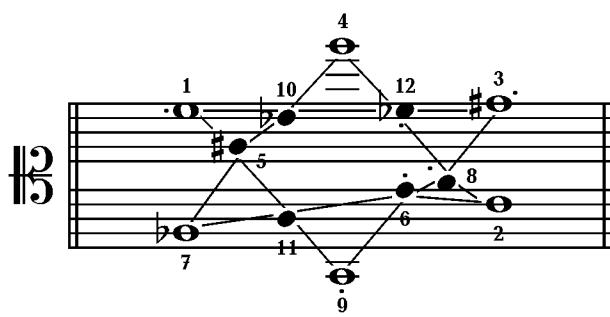
(1)



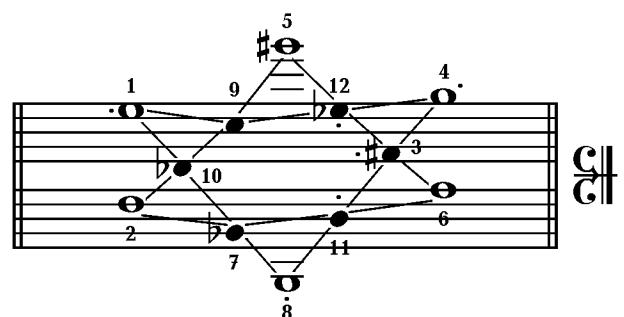
(2)



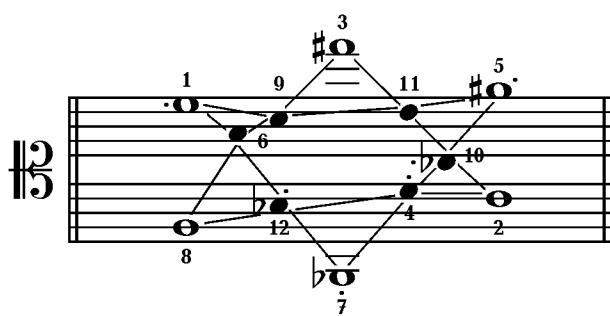
(3)



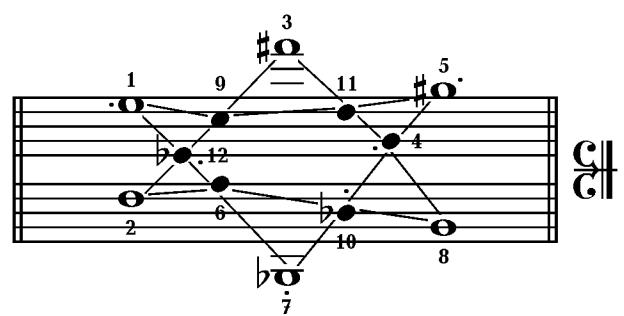
(4)



(5)



(6)

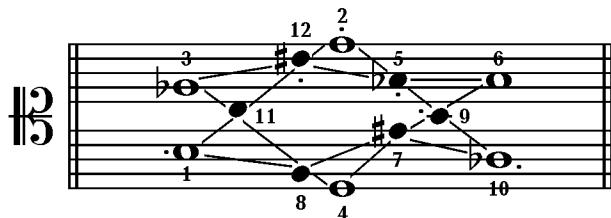


...Horizontal Stars

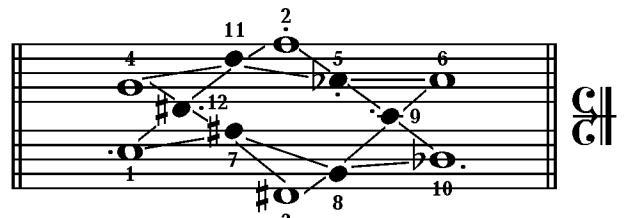
VI



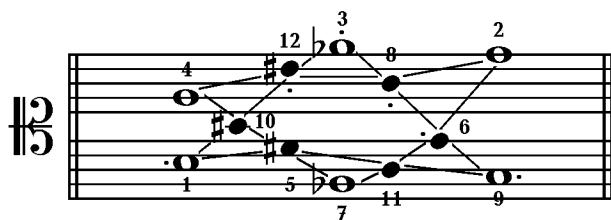
(1)



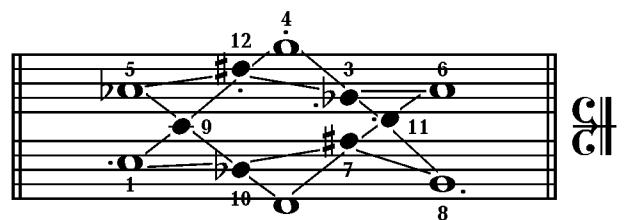
(2)



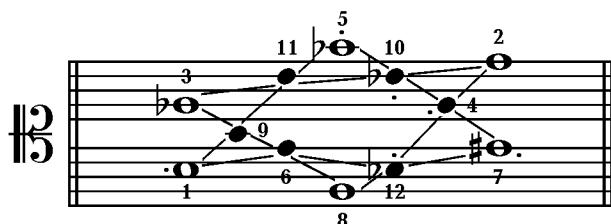
(3)



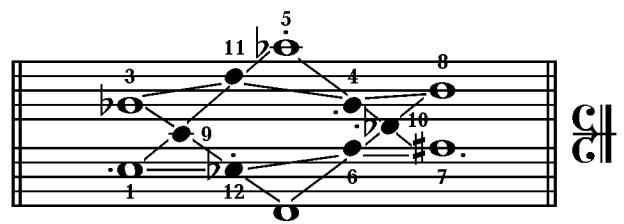
(4)



(5)



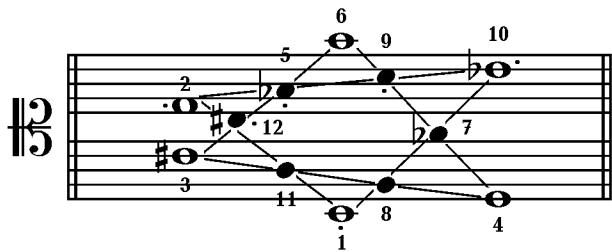
(6)



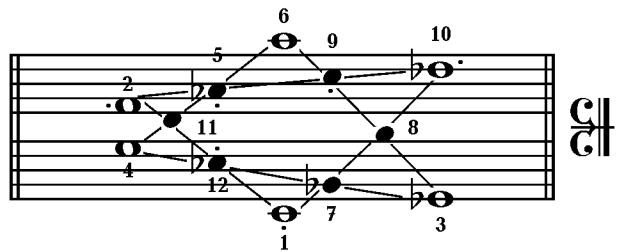
Horizontal Stars Sloped...

I

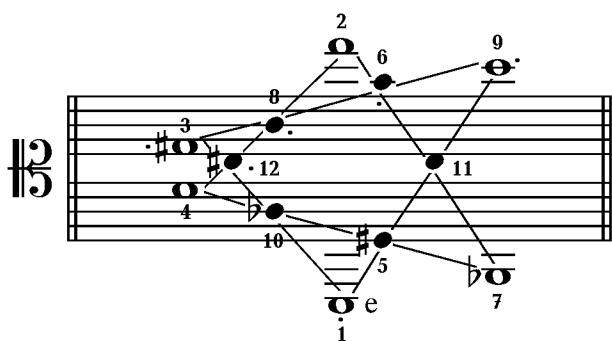
(1)



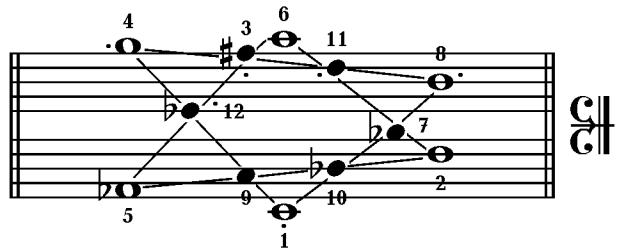
(2)



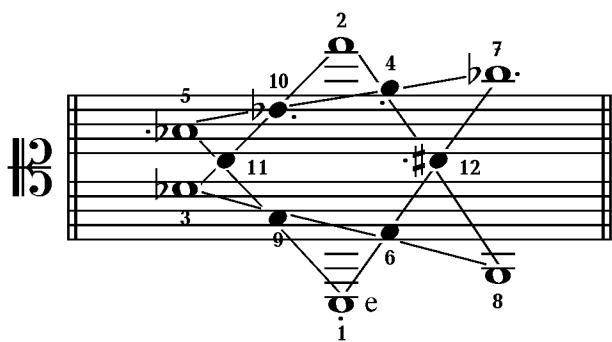
(3)



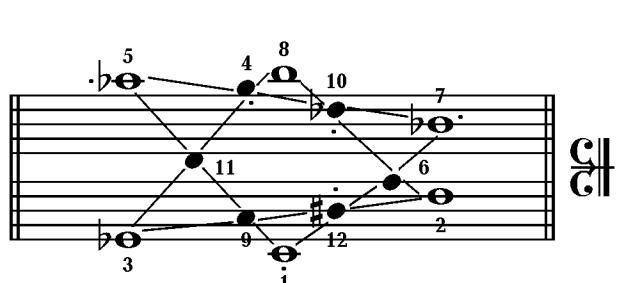
(4)



(5)



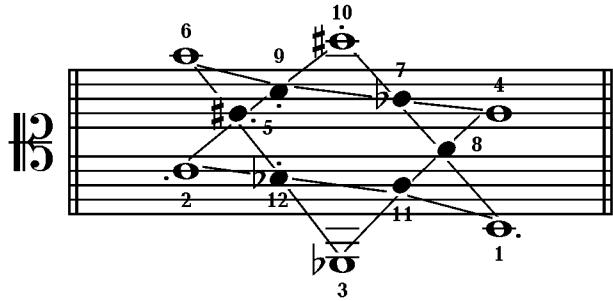
(6)



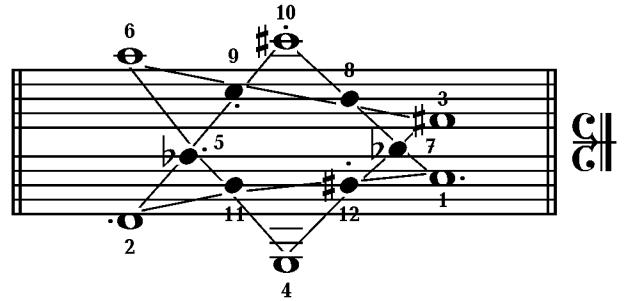
...Horizontal Stars Sloped...

II

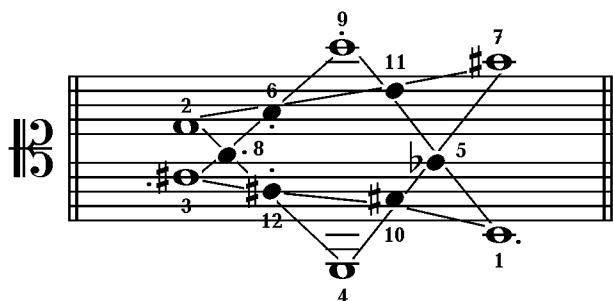
(1)



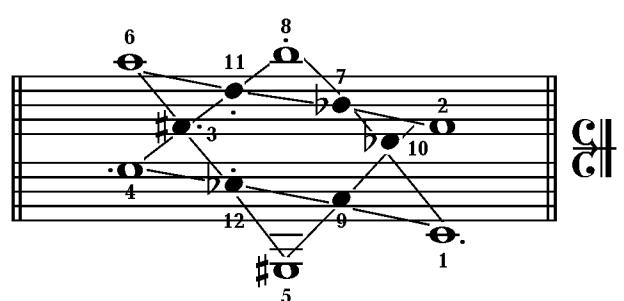
(2)



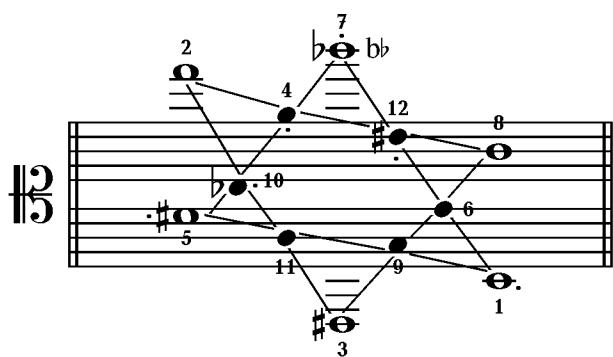
(3)



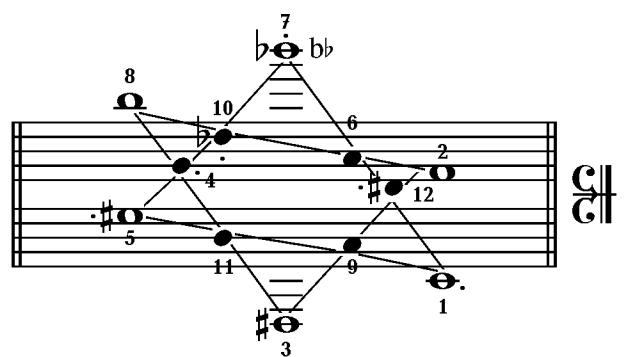
(4)



(5)



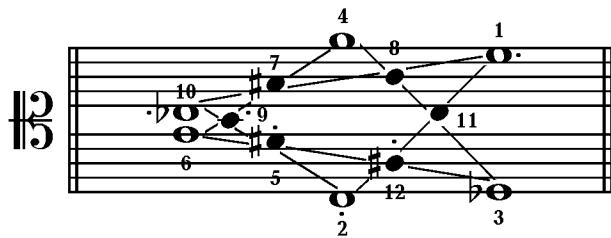
(6)



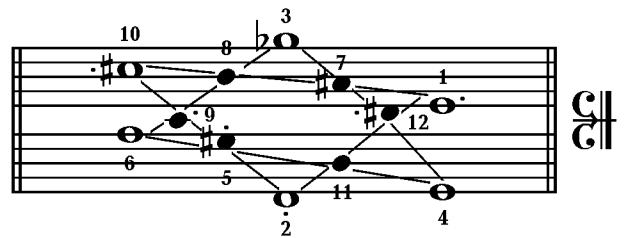
...Horizontal Stars Sloped...

III

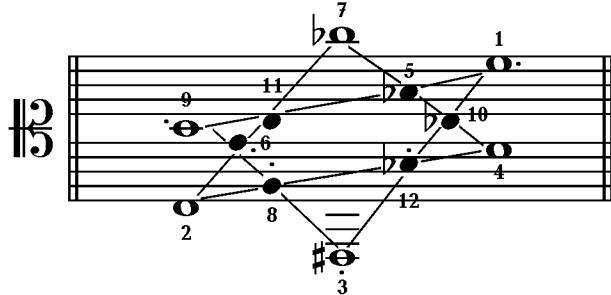
(1)



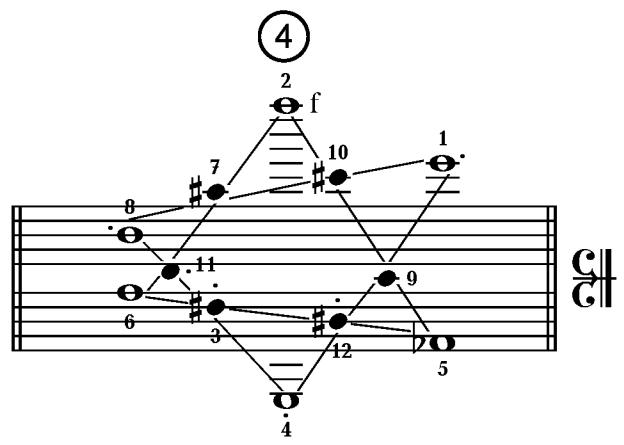
(2)



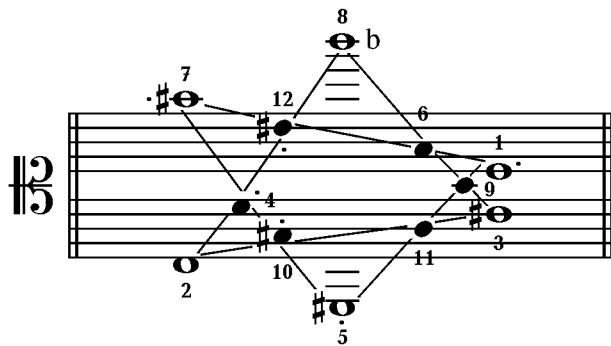
(3)



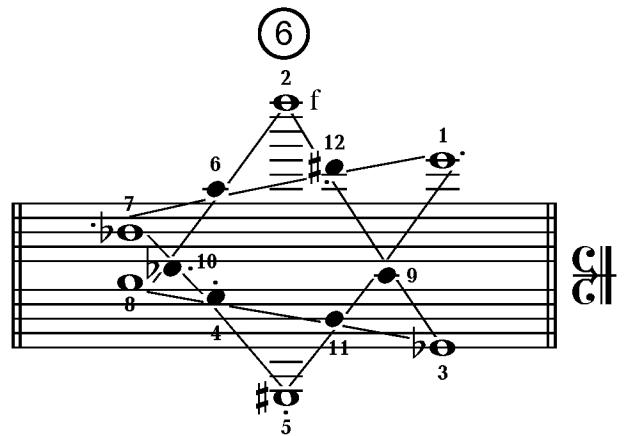
(4)



(5)



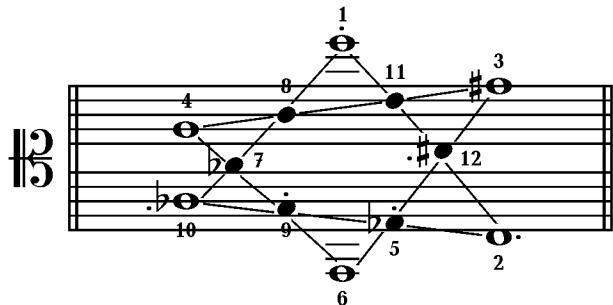
(6)



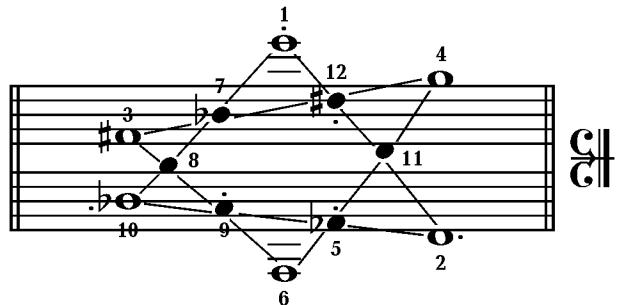
...Horizontal Stars Sloped...

IV

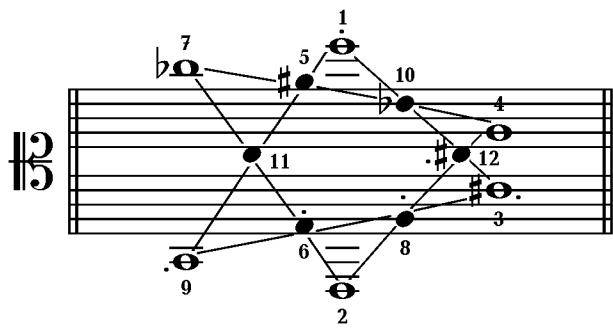
(1)



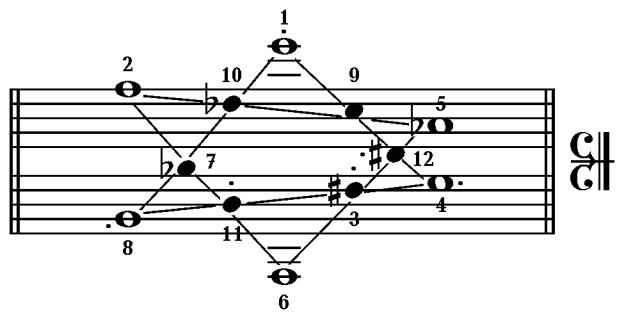
(2)



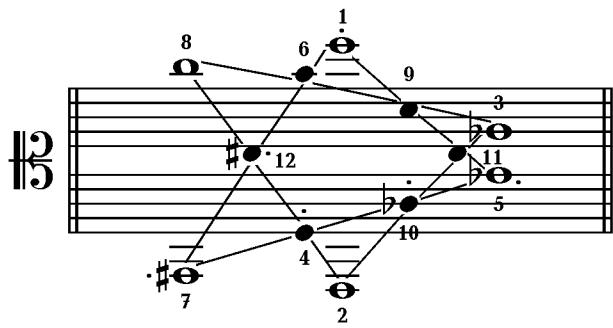
(3)



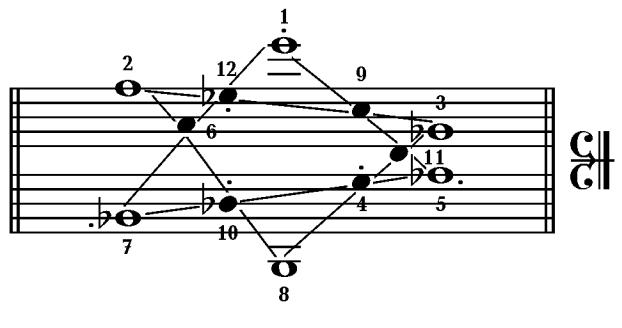
(4)



(5)



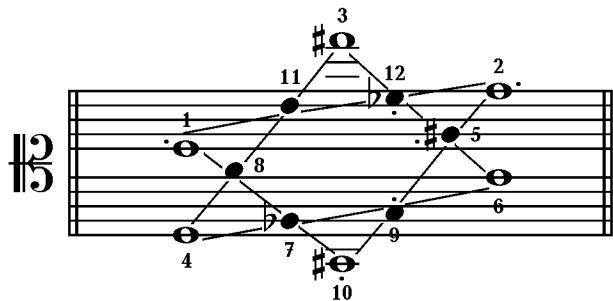
(6)



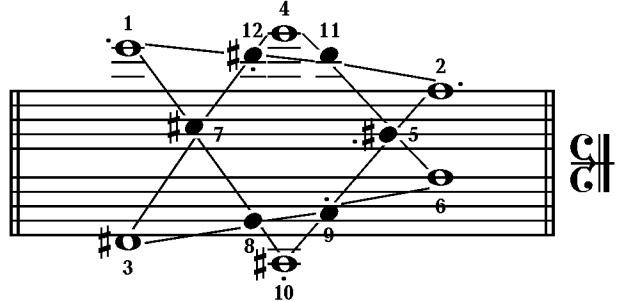
...Horizontal Stars Sloped...

V

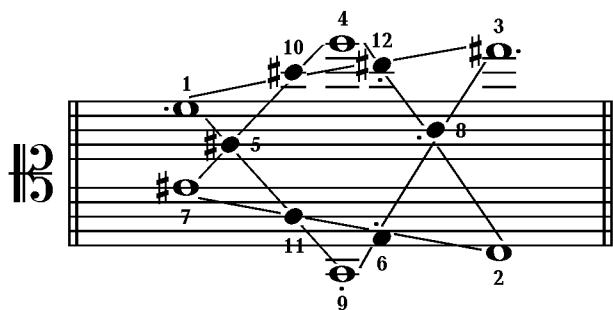
(1)



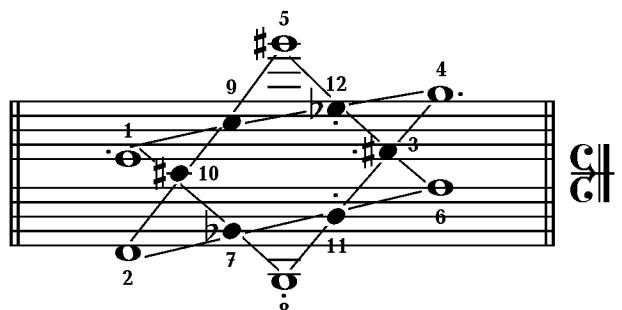
(2)



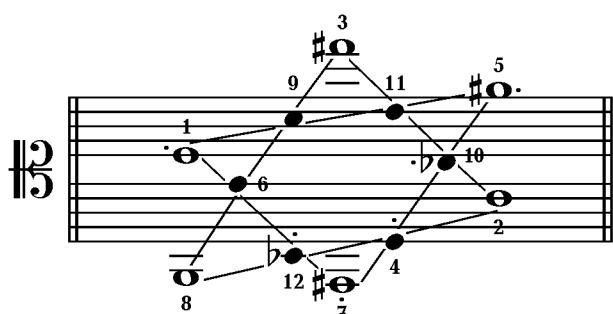
(3)



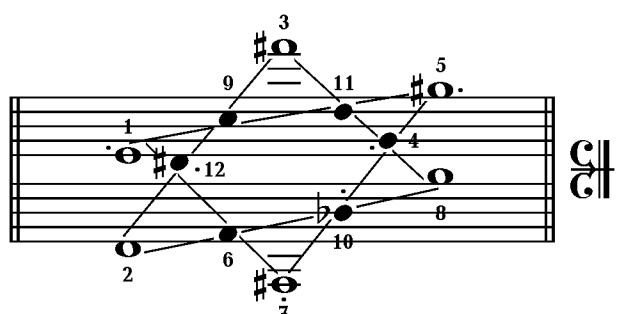
(4)



(5)



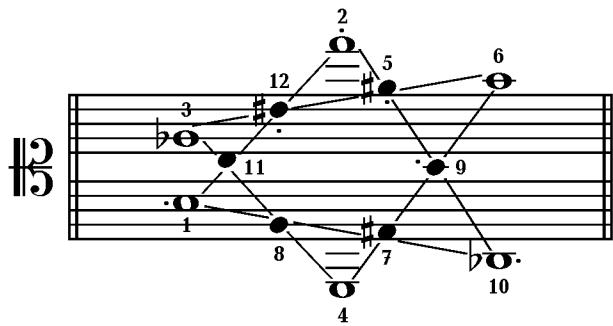
(6)



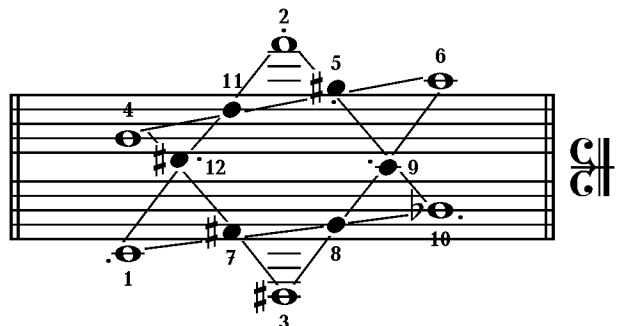
...Horizontal Stars Sloped

VI

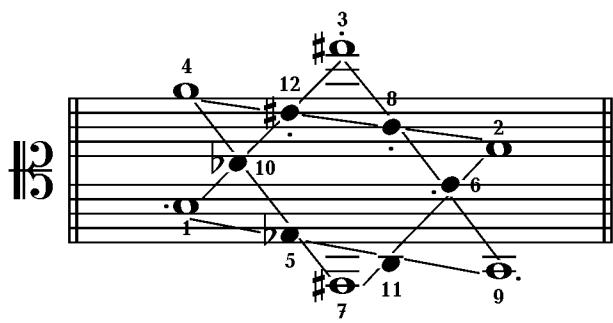
(1)



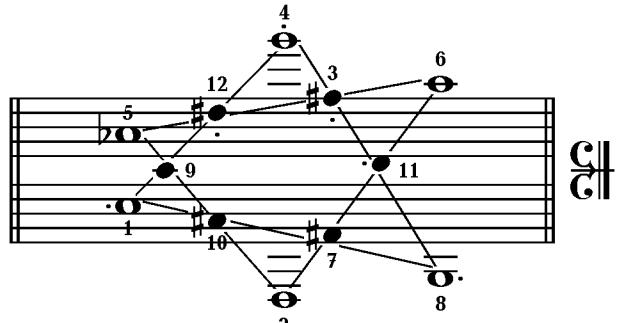
(2)



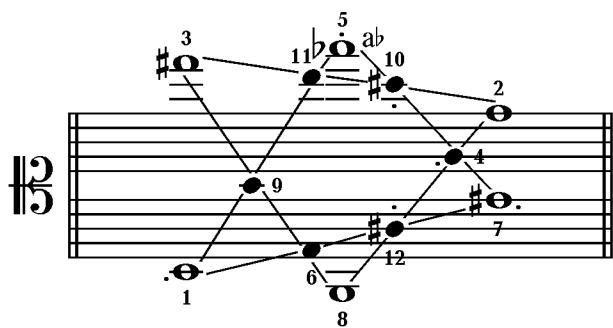
(3)



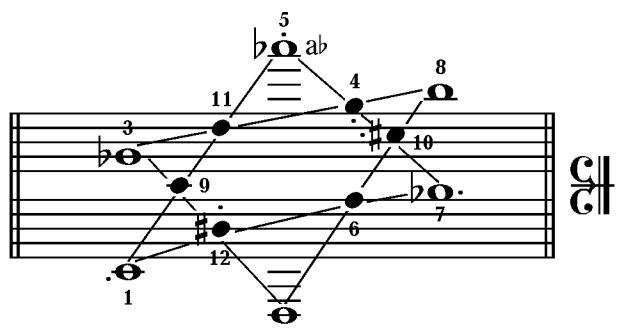
(4)



(5)



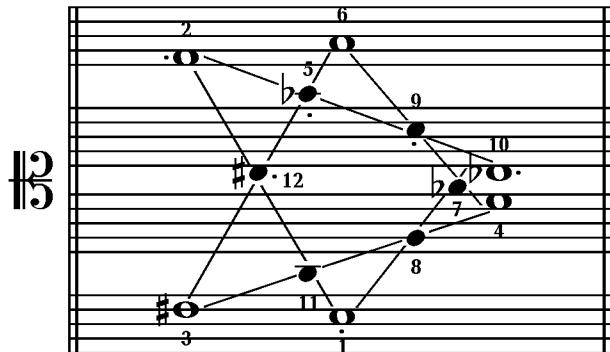
(6)



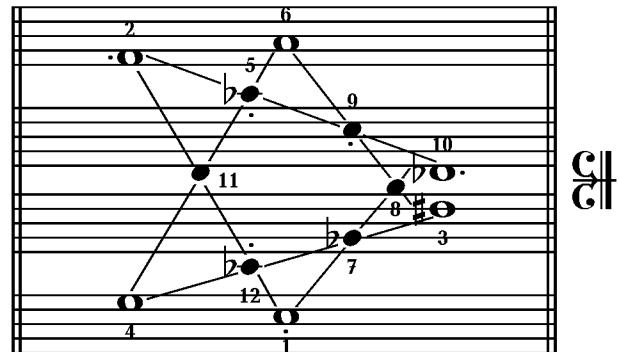
Horizontal Stars Extra Sloped...

I

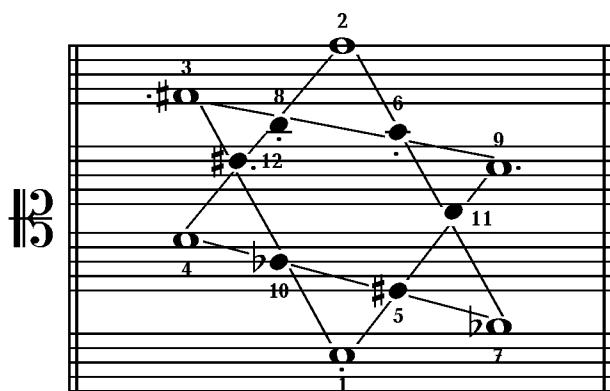
(1)



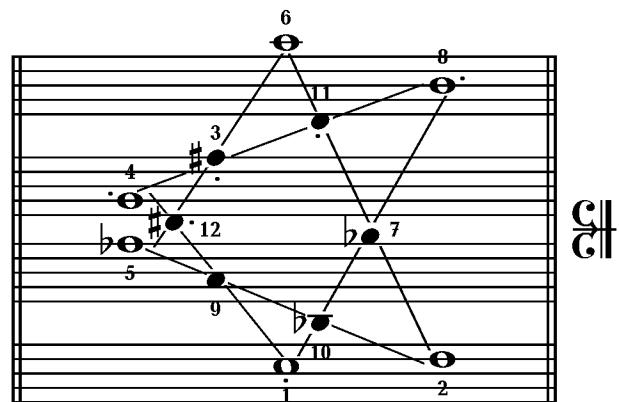
(2)



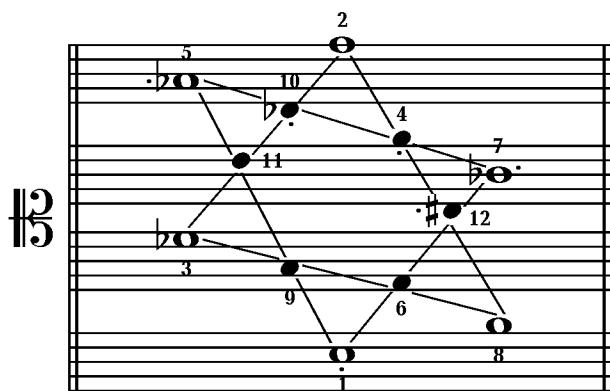
(3)



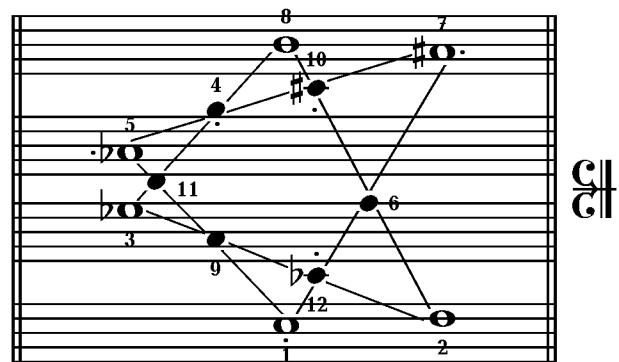
(4)



(5)



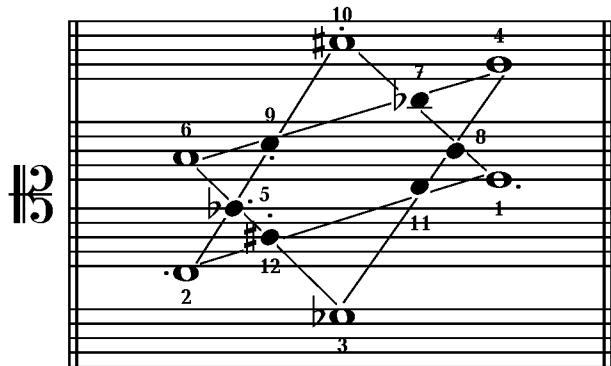
(6)



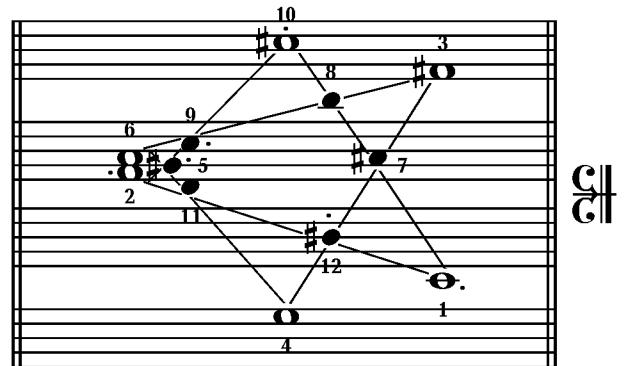
...Horizontal Stars Extra Sloped...

II

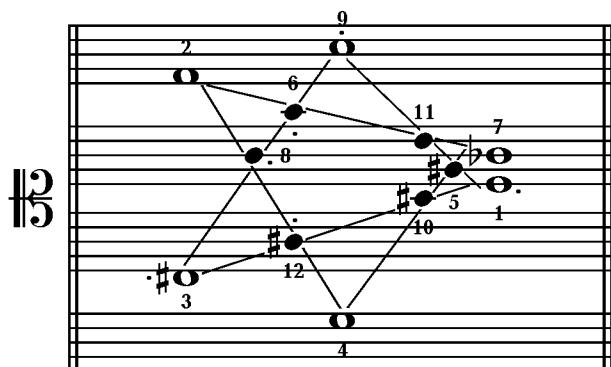
(1)



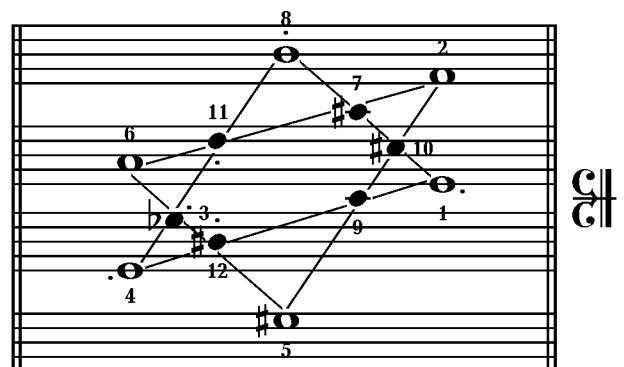
(2)



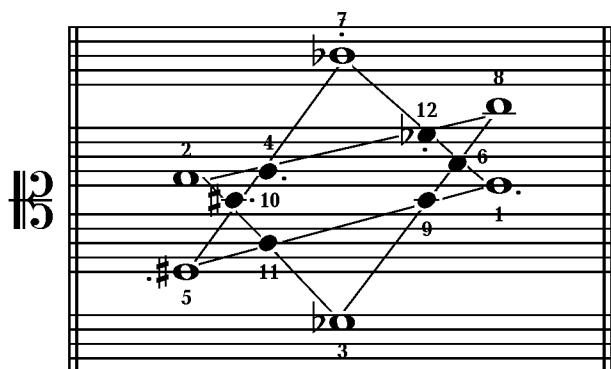
(3)



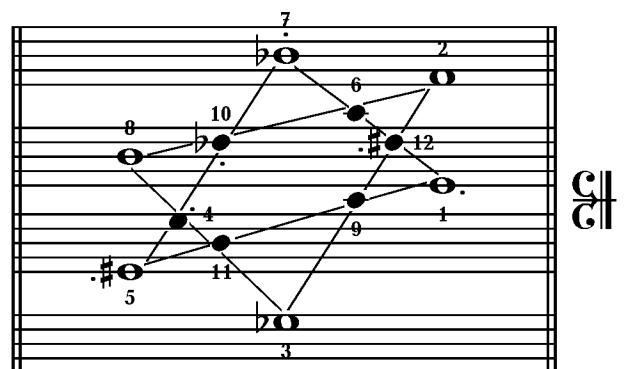
(4)



(5)



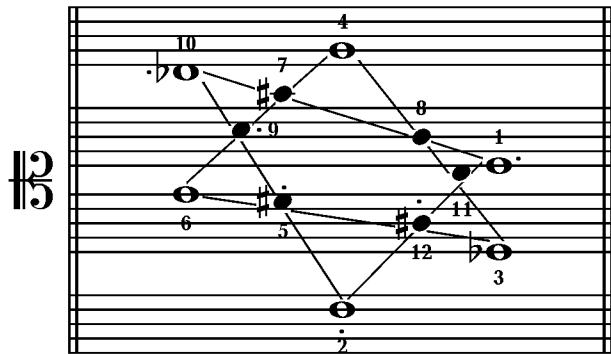
(6)



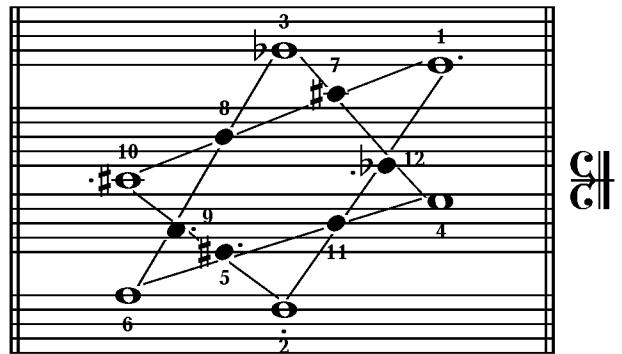
...Horizontal Stars Extra Sloped...

III

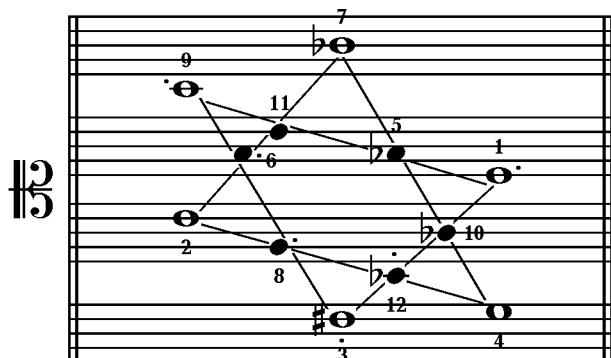
(1)



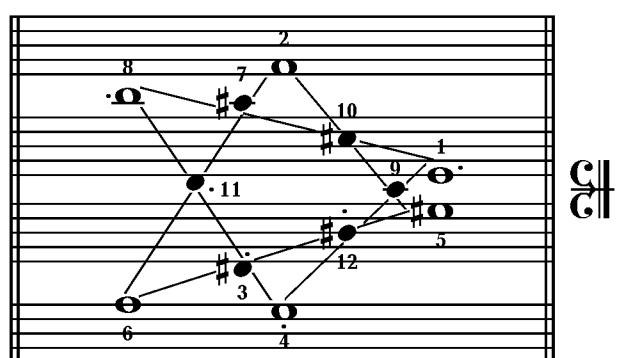
(2)



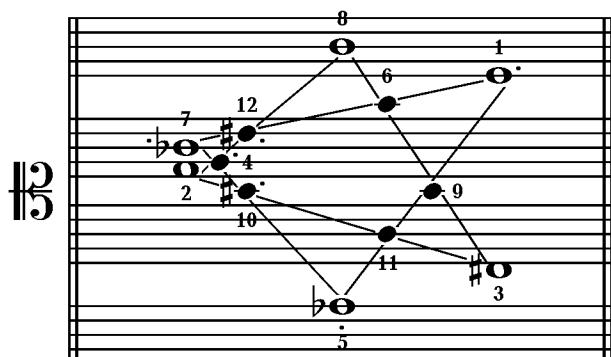
(3)



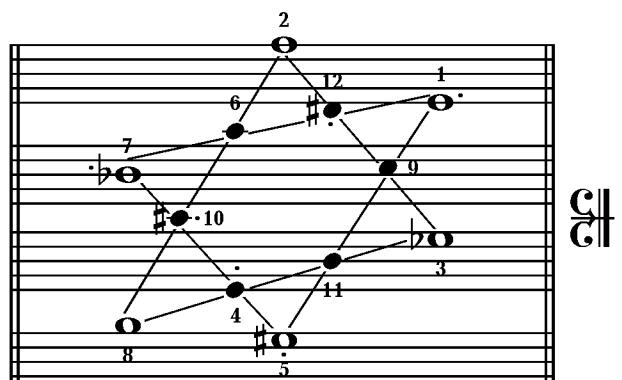
(4)



(5)



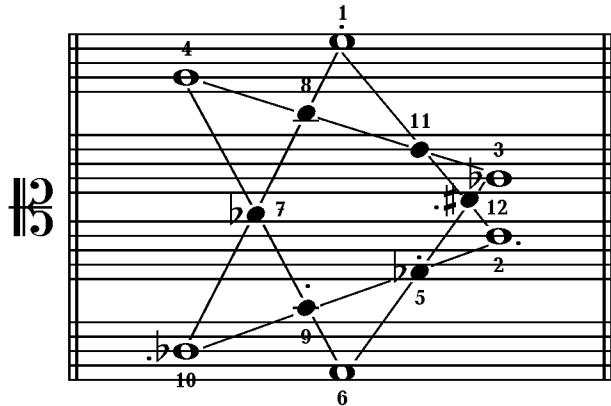
(6)



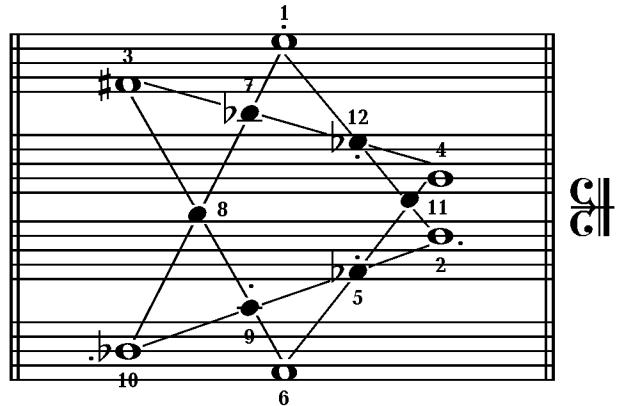
...Horizontal Stars Extra Sloped...

IV

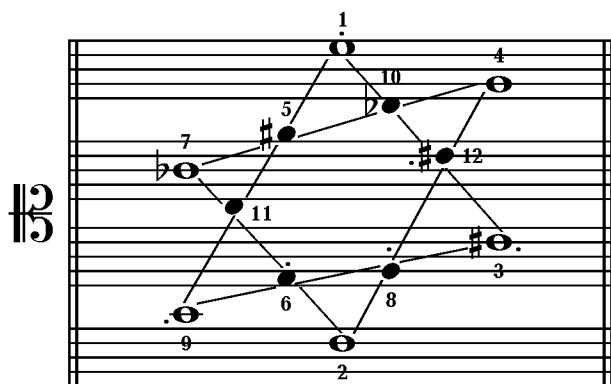
(1)



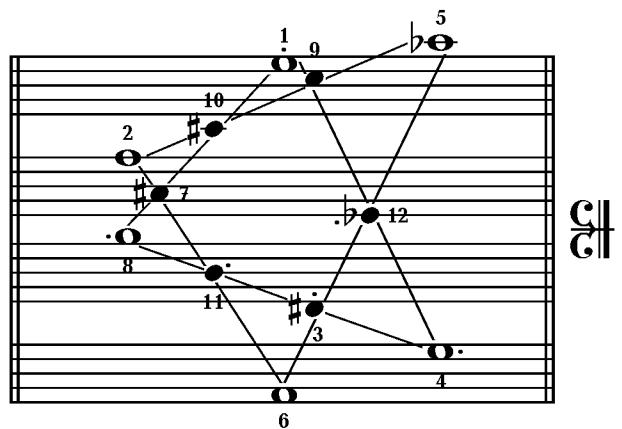
(2)



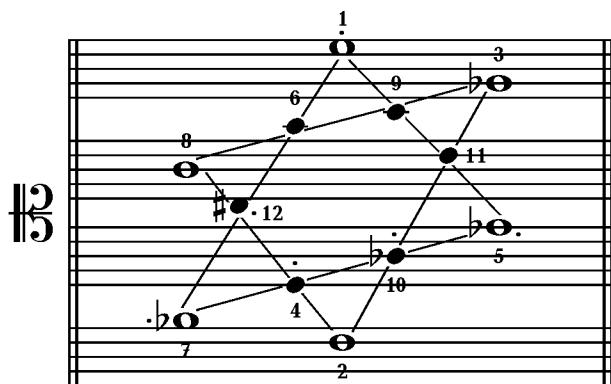
(3)



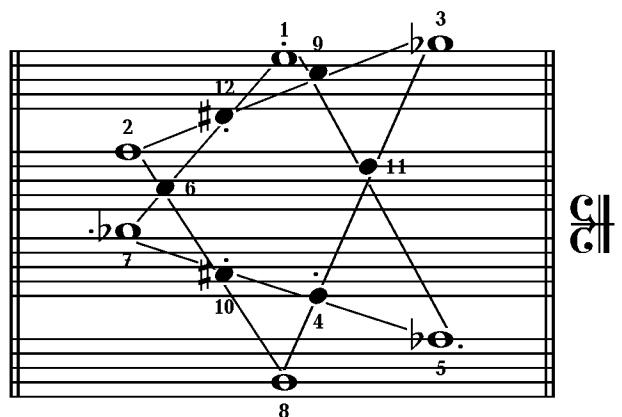
(4)



(5)



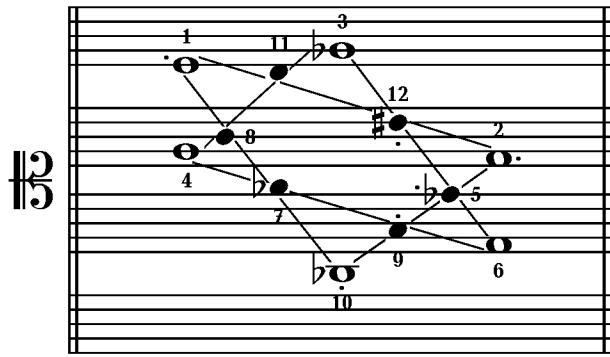
(6)



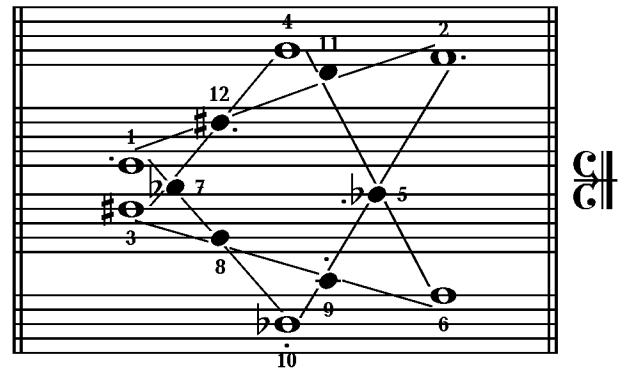
...Horizontal Stars Extra Sloped...

V

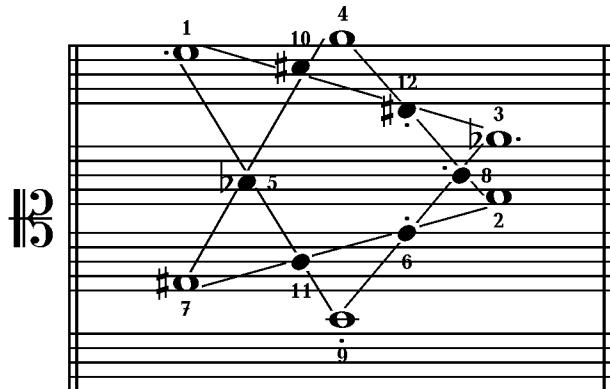
(1)



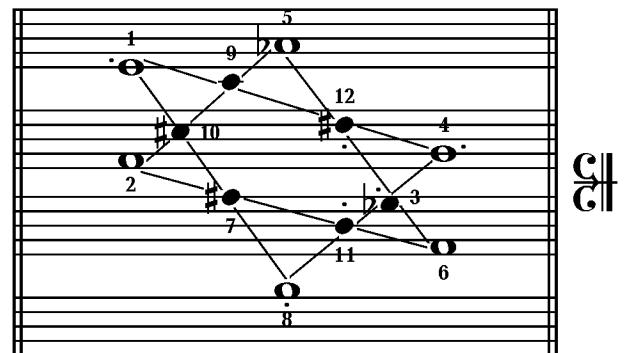
(2)



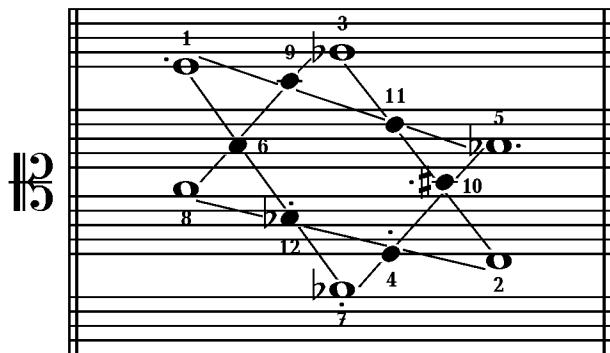
(3)



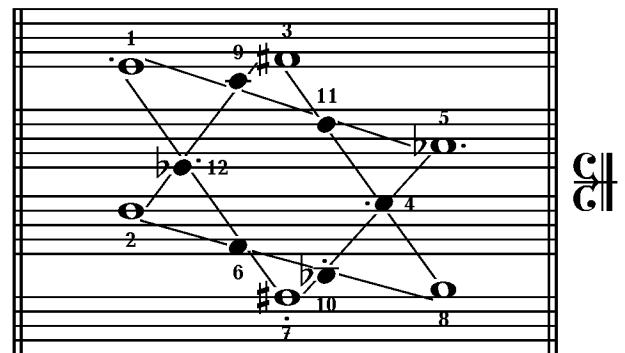
(4)



(5)



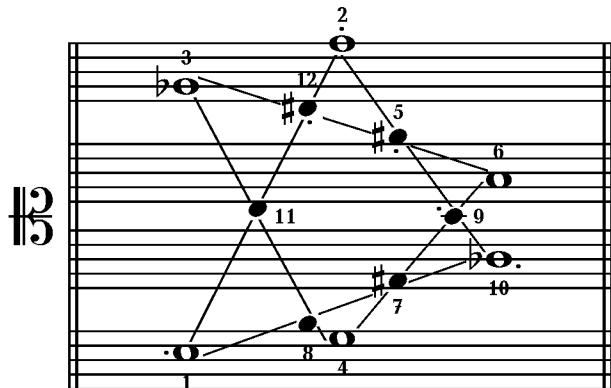
(6)



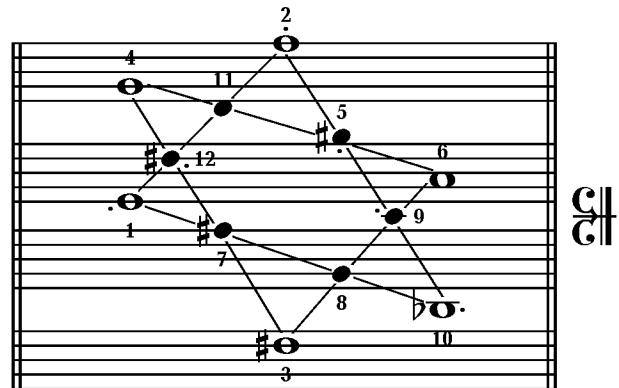
...Horizontal Stars Extra Sloped

VI

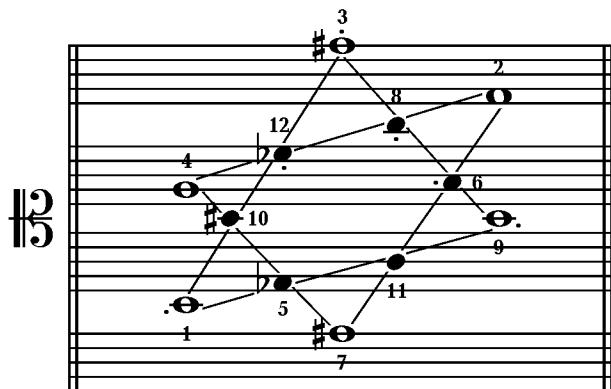
(1)



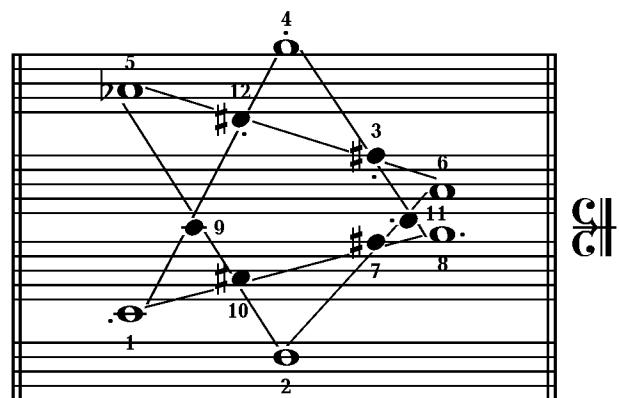
(2)



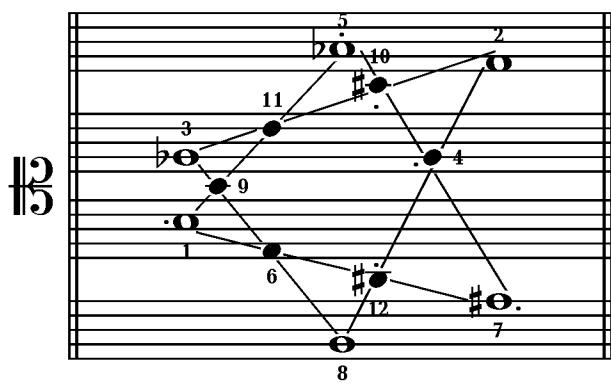
(3)



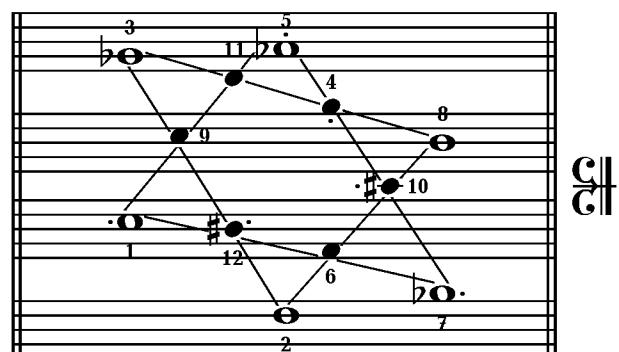
(4)



(5)



(6)



Horizontal Stars Compressed...

I

Diagram I consists of six numbered diagrams (1-6) showing compressed horizontal stars. Each diagram is a 12-note chromatic scale on a staff with various notes highlighted by dots and connected by dashed lines. The diagrams illustrate different ways to compress the scale into six notes.

II

Diagram II consists of six numbered diagrams (1-6) showing compressed horizontal stars. These diagrams are similar to those in Diagram I, illustrating different ways to compress the scale into six notes.

III

Diagram III consists of six numbered diagrams (1-6) showing compressed horizontal stars. These diagrams are similar to those in Diagrams I and II, illustrating different ways to compress the scale into six notes.

...Horizontal Stars Compressed

IV

Diagram showing six horizontal star patterns for the IV mode. Each diagram consists of a 5-line staff with numbered dots (1-12) indicating note positions. The patterns are labeled 1 through 6.

- Diagram 1:** Shows a star shape with points at 1, 11, 3, 12, 4, 2, 10, 9, 6, and 8.
- Diagram 2:** Shows a star shape with points at 1, 12, 4, 7, 8, 3, 10, 9, 6, and 2.
- Diagram 3:** Shows a star shape with points at 1, 10, 5, 7, 11, 2, 9, 6, 8, and 3.
- Diagram 4:** Shows a star shape with points at 1, 9, 5, 10, 12, 6, 2, 11, 3, and 4.
- Diagram 5:** Shows a star shape with points at 1, 9, 3, 8, 12, 6, 2, 10, 4, and 2.
- Diagram 6:** Shows a star shape with points at 1, 9, 3, 11, 12, 6, 4, 10, 8, and 5.

5

Diagram showing six horizontal star patterns for the V mode. Each diagram consists of a 5-line staff with numbered dots (1-12) indicating note positions. The patterns are labeled 1 through 6.

- Diagram 1:** Shows a star shape with points at 1, 3, 12, 2, 4, 11, 8, 5, 7, and 10.
- Diagram 2:** Shows a star shape with points at 1, 4, 2, 12, 11, 3, 5, 7, 8, and 6.
- Diagram 3:** Shows a star shape with points at 1, 4, 3, 10, 12, 5, 2, 6, 8, and 7.
- Diagram 4:** Shows a star shape with points at 1, 5, 12, 9, 11, 4, 2, 10, 7, and 8.
- Diagram 5:** Shows a star shape with points at 1, 3, 11, 10, 12, 5, 4, 9, 7, and 6.
- Diagram 6:** Shows a star shape with points at 1, 3, 11, 12, 4, 5, 2, 10, 8, and 7.

V

Diagram showing six horizontal star patterns for the VI mode. Each diagram consists of a 5-line staff with numbered dots (1-12) indicating note positions. The patterns are labeled 1 through 6.

- Diagram 1:** Shows a star shape with points at 1, 2, 12, 5, 6, 3, 11, 4, 7, and 10.
- Diagram 2:** Shows a star shape with points at 1, 2, 5, 11, 12, 4, 3, 6, 7, and 8.
- Diagram 3:** Shows a star shape with points at 1, 2, 3, 10, 12, 5, 4, 6, 8, and 7.
- Diagram 4:** Shows a star shape with points at 1, 5, 12, 9, 11, 2, 3, 10, 7, and 6.
- Diagram 5:** Shows a star shape with points at 1, 3, 11, 10, 12, 5, 4, 9, 7, and 6.
- Diagram 6:** Shows a star shape with points at 1, 3, 11, 12, 4, 5, 2, 10, 8, and 7.

VI

Diagram showing six horizontal star patterns for the VII mode. Each diagram consists of a 5-line staff with numbered dots (1-12) indicating note positions. The patterns are labeled 1 through 6.

- Diagram 1:** Shows a star shape with points at 12, 2, 5, 6, 3, 11, 4, 7, and 10.
- Diagram 2:** Shows a star shape with points at 11, 2, 5, 12, 1, 3, 4, 8, and 10.
- Diagram 3:** Shows a star shape with points at 4, 2, 3, 10, 12, 5, 6, 8, and 7.
- Diagram 4:** Shows a star shape with points at 5, 4, 12, 9, 11, 2, 3, 10, 7, and 6.
- Diagram 5:** Shows a star shape with points at 3, 5, 11, 10, 12, 2, 4, 9, 7, and 6.
- Diagram 6:** Shows a star shape with points at 3, 5, 11, 12, 4, 10, 2, 8, and 7.

Horizontal Stars Degenerated...

I

Figure I consists of six staves, each containing a single horizontal line of notes. The notes are connected by vertical stems pointing downwards. The patterns are as follows:

- Staff 1: Notes 2, 5, 6, 9, 7, 10.
- Staff 2: Notes 2, 5, 6, 9, 8, 10.
- Staff 3: Notes 3, 12, 8, 6, 9, 11.
- Staff 4: Notes 4, 12, 3, 6, 11, 7, 8.
- Staff 5: Notes 5, 10, 2, 4, 12, 7.
- Staff 6: Notes 5, 4, 8, 10, 6, 7.

(4)

Figure I continues with staves 4 through 6:

- Staff 4: Notes 4, 12, 3, 6, 11, 7, 8.
- Staff 5: Notes 5, 10, 2, 4, 12, 7.
- Staff 6: Notes 5, 4, 8, 10, 6, 7.

II

Figure II consists of six staves, each containing a single horizontal line of notes. The patterns are as follows:

- Staff 1: Notes 6, 5, 9, 10, 7, 4.
- Staff 2: Notes 6, 5, 9, 10, 8, 7, 3.
- Staff 3: Notes 2, 6, 9, 11, 7.
- Staff 4: Notes 6, 3, 11, 8, 7, 2.
- Staff 5: Notes 2, 4, 10, 7, 12, 6, 8.
- Staff 6: Notes 8, 4, 10, 7, 6, 12.

(4)

Figure II continues with staves 4 through 6:

- Staff 4: Notes 6, 3, 11, 8, 7, 2.
- Staff 5: Notes 2, 4, 10, 7, 12, 6, 8.
- Staff 6: Notes 8, 4, 10, 7, 6, 12.

III

Figure III consists of six staves, each containing a single horizontal line of notes. The patterns are as follows:

- Staff 1: Notes 10, 9, 7, 4, 8.
- Staff 2: Notes 10, 9, 8, 7, 12, 1.
- Staff 3: Notes 9, 6, 11, 5, 10, 1.
- Staff 4: Notes 8, 10, 7, 2, 10, 9, 1.
- Staff 5: Notes 7, 12, 8, 6, 9, 1.
- Staff 6: Notes 7, 10, 2, 12, 9, 1.

(4)

Figure III continues with staves 4 through 6:

- Staff 4: Notes 8, 10, 7, 2, 10, 9, 1.
- Staff 5: Notes 7, 12, 8, 6, 9, 1.
- Staff 6: Notes 7, 10, 2, 12, 9, 1.

...Horizontal Stars Degenerated

IV

Figure showing six musical staves (1 through 6) for the IV section. Each staff consists of five horizontal lines representing a staff and vertical stems extending downwards. The notes are represented by small circles with numbers indicating their pitch. The staves are arranged in two rows of three. The first row contains staves 1, 2, and 3. The second row contains staves 4, 5, and 6. The key signature is one sharp (F#), and the time signature is common time (indicated by a 'C').

V

Figure showing six musical staves (1 through 6) for the V section. The arrangement is identical to the IV section, with staves 1, 2, and 3 in the top row and staves 4, 5, and 6 in the bottom row. The key signature changes to two sharps (G), and the time signature remains common time.

VI

Figure showing six musical staves (1 through 6) for the VI section. The arrangement is identical to the previous sections. The key signature changes to three sharps (D), and the time signature remains common time.

Sides 1

(Horizontal Hexads in 4)

(1)

1₁ 1₃ 1₅2₅ 1₇ 1₉ 1₁₁
10 6.2 1.3 2.2 5.2 1.1

(2)

2₁ 2₃ 1₅2₅ 2₇ 2₉ 2₁₁
10 8 1.3 6.3 5.2 4.2

(3)

3₁ 3₃ 3₅ 3₇ 3₉ 3₁₁
2.1 7.3 6.3 1.2 7.2 5.1

(4)

4₁ 4₃ 4₅ 4₇ 4₉ 4₁₁
4.1 6.2 1.1 1.2 5.2 1.3

(5)

5₁6₁ 5₃ 5₅ 5₇ 5₉ 5₁₁
7.2 3.1 5.1 7.3 9 6.3

(6)

5₁6₁ 6₃ 6₅ 6₇ 6₉ 6₁₁
7.2 1.1 5.1 7.1 9 6.1

Sides 2

(Horizontal Hexads in 4)

(1)

1¹ 1³ 1⁵2⁵ 1⁷ 1⁹ 1¹¹
10 6.2 1.3 2.2 5.2 1.1

(2)

2¹ 2³ 1⁵2⁵ 2⁷ 2⁹ 2¹¹
10 8 1.3 6.3 5.2 4.2

(3)

3¹ 3³ 3⁵ 3⁷ 3⁹ 3¹¹
2.1 7.3 6.3 1.2 7.2 5.1

(4)

4¹ 4³ 4⁵ 4⁷ 4⁹ 4¹¹
4.1 6.2 1.1 1.2 5.2 1.3

(5)

5¹6¹ 5³ 5⁵ 5⁷ 5⁹ 5¹¹
7.2 3.1 5.1 7.3 9 6.3

(6)

5¹6¹ 6³ 6⁵ 6⁷ 6⁹ 6¹¹
7.2 1.1 5.1 7.1 9 6.1

Sides 3

(1)

1_1 1^3 $1_5 2_5$ 1^7 1_9 1^{11}
10 6.2 1.3 2.2 5.2 1.1

(2)

2_1 2^3 $1_5 2_5$ 2^7 2_9 2^{11}
10 8 1.3 6.3 5.2 4.2

(3)

3_1 3^3 3_5 3^7 3_9 3^{11}
2.1 7.3 6.3 1.2 7.2 5.1

(4)

4_1 4^3 4_5 4^7 4_9 4^{11}
4.1 6.2 1.1 1.2 5.2 1.3

(5)

$5_1 6_1$ 5^3 5_5 5^7 5_9 5^{11}
7.2 3.1 5.1 7.3 9 6.3

(6)

$5_1 6_1$ 6^3 6_5 6^7 6_9 6^{11}
7.2 1.1 5.1 7.1 9 6.1

Sides 4

(1)

1^1 1_3 $1^{52} 5$ 1_7 1^9 1_{11}
10 6.2 1.3 2.2 5.2 1.1

(2)

2_1 2_3 $1^{52} 5$ 2_7 2^9 2_{11}
10 8 1.3 6.3 5.2 4.2

(3)

3_1 3_3 3^5 3_7 3^9 3_{11}
2.1 7.3 6.3 1.2 7.2 5.1

(4)

4_1 4_3 4^5 4_7 4^9 4_{11}
4.1 6.2 1.1 1.2 5.2 1.3

(5)

$5^1 6^1$ 5_3 5^5 5_7 5^9 5_{11}
7.2 3.1 5.1 7.3 9 6.3

(6)

$5^1 6^1$ 6_3 6^5 6_7 6^9 6_{11}
7.2 1.1 5.1 7.1 9 6.1

Sides of Triangles 1

▽ (1) △

1₁ 1₅2₅ 1₉ 1₃ 1₇ 1₁₁
10 1.3 5.2 6.2 2.2 1.1

▽ (2) △

2₁ 1₅2₅ 2₉ 2₃ 2₇ 2₁₁
10 1.3 5.2 8 6.3 4.2

▽ (3) △

3₁ 3₅ 3₉ 3₃ 3₇ 3₁₁
2.1 6.3 7.2 7.3 1.2 5.1

▽ (4) △

4₁ 4₅ 4₉ 4₃ 4₇ 4₁₁
4.1 1.1 5.2 6.2 1.2 1.3

▽ (5) △

5₁6₁ 5₅ 5₉ 5₃ 5₇ 5₁₁
7.2 5.1 9 3.1 7.3 6.3

▽ (6) △

5₁6₁ 6₅ 6₉ 6₃ 6₇ 6₁₁
7.2 5.1 9 1.1 7.1 6.1

Sides of Triangles 2

▽ (1) △

1¹ 1⁵2⁵ 1⁹ 1³ 1⁷ 1¹¹
10 1.3 5.2 6.2 2.2 1.1

▽ (2) △

2¹ 1⁵2⁵ 2⁹ 2³ 2⁷ 2¹¹
10 1.3 5.2 8 6.3 4.2

▽ (3) △

3¹ 3⁵ 3⁹ 3³ 3⁷ 3¹¹
2.1 6.3 7.2 7.3 1.2 5.1

▽ (4) △

4¹ 4⁵ 4⁹ 4³ 4⁷ 4¹¹
4.1 1.1 5.2 6.2 1.2 1.3

▽ (5) △

5¹6¹ 5⁵ 5⁹ 5³ 5⁷ 5¹¹
7.2 5.1 9 3.1 7.3 6.3

▽ (6) △

5¹6¹ 6⁵ 6⁹ 6³ 6⁷ 6¹¹
7.2 5.1 9 1.1 7.1 6.1

Sides of Downward Triangles (∇)

(1)

1₁ 1⁵2⁵ 1₉ 1¹ 1₅2₅ 1⁹
10 1.3 5.2 10 1.3 5.2

(2)

2₁ 1⁵2⁵ 2₉ 2¹ 1₅2₅ 2⁹
10 1.3 5.2 10 1.3 5.2

(3)

3₁ 3⁵ 3₉ 3¹ 3₅ 3⁹
2.1 6.3 7.2 2.1 6.3 7.2

(4)

4₁ 4⁵ 4₉ 4¹ 4₅ 4⁹
4.1 1.1 5.2 4.1 1.1 5.2

(5)

5₁6₁ 5⁵ 5₉ 5¹6₁ 5₅ 5⁹
7.2 5.1 9 7.2 5.1 9

(6)

5₁6₁ 6⁵ 6₉ 5¹6₁ 6₅ 6⁹
7.2 5.1 9 7.2 5.1 9

Sides of Upward Triangles (Δ)

(1)

1₃ 1⁷ 1₁₁ 1³ 1₇ 1¹¹
6.2 2.2 1.1 6.2 2.2 1.1

(2)

2₃ 2⁷ 2₁₁ 2³ 2₇ 2¹¹
8 6.3 4.2 8 6.3 4.2

(3)

3₃ 3⁷ 3₁₁ 3³ 3₇ 3¹¹
7.3 1.2 5.1 7.3 1.2 5.1

(4)

4₃ 4⁷ 4₁₁ 4³ 4₇ 4¹¹
6.2 1.2 1.3 6.2 1.2 1.3

(5)

5₃ 5⁷ 5₁₁ 5³ 5₇ 5¹¹
3.1 7.3 6.3 3.1 7.3 6.3

(6)

6₃ 6⁷ 6₁₁ 6³ 6₇ 6¹¹
1.1 7.1 6.1 1.1 7.1 6.1

Opposite Sides 1

(1) (2) (3) I (4) (5) (6)

1₁ 1₇ 2₁ 2₇ 3₁ 3₇ 4₁ 4₇ 5₁6₁ 5₇ 5₁6₁ 6₇
10 2.2 10 6.3 2.1 1.2 4.1 1.2 7.2 7.3 7.2 7.1

(1) (2) (3) II (4) (5) (6)

1₃ 1₉ 2₃ 2₉ 3₃ 3₉ 4₃ 4₉ 5₃ 5₉ 6₃ 6₉
6.2 5.2 8 5.2 7.3 7.2 6.2 5.2 3.1 9 1.1 9

(1) (2) (3) III (4) (5) (6)

1₅2₅ 1₁₁ 1₅2₅ 2₁₁ 3₅ 3₁₁ 4₅ 4₁₁ 5₅ 5₁₁ 6₅ 6₁₁
1.3 1.1 1.3 4.2 6.3 5.1 1.1 1.3 5.1 6.3 5.1 6.1

(1) (2) (3) IV (4) (5) (6)

1₇ 1₁ 2₇ 2₁ 3₇ 3₁ 4₇ 4₁ 5₇ 5₁6₁ 6₇ 5₁6₁
2.2 10 6.3 10 1.2 2.1 1.2 4.1 7.3 7.2 7.1 7.2

(1) (2) (3) V (4) (5) (6)

1₉ 1₃ 2₉ 2₃ 3₉ 3₃ 4₉ 4₃ 5₉ 5₃ 6₉ 6₃
5.2 6.2 5.2 8 7.2 7.3 5.2 6.2 9 3.1 9 1.1

(1) (2) (3) VI (4) (5) (6)

1₁₁ 1₅2₅ 2₁₁ 1₅2₅ 3₁₁ 3₅ 4₁₁ 4₅ 5₁₁ 5₅ 6₁₁ 6₅
1.1 1.3 4.2 1.3 5.1 6.3 1.3 1.1 6.3 5.1 6.1 5.1

Opposite Sides 2

(1) (2) (3) I (4) (5) (6)

1¹ 1⁷ 2¹ 2⁷ 3¹ 3⁷ 4¹ 4⁷ 5¹6¹ 5⁷ 5¹6¹ 6⁷
10 2.2 10 6.3 2.1 1.2 4.1 1.2 7.2 7.3 7.2 7.1

(1) (2) (3) II (4) (5) (6)

1³ 1⁹ 2³ 2⁹ 3³ 3⁹ 4³ 4⁹ 5³ 5⁹ 6³ 6⁹
6.2 5.2 8 5.2 7.3 7.2 6.2 5.2 3.1 9 1.1 9

(1) (2) (3) III (4) (5) (6)

1⁵2⁵ 1¹¹ 1⁵2⁵ 2¹¹ 3⁵ 3¹¹ 4⁵ 4¹¹ 5⁵ 5¹¹ 6⁵ 6¹¹
1.3 1.1 1.3 4.2 6.3 5.1 1.1 1.3 5.1 6.3 5.1 6.1

(1) (2) (3) IV (4) (5) (6)

1⁷ 1¹ 2⁷ 2¹ 3⁷ 3¹ 4⁷ 4¹ 5⁷ 5¹6¹ 6⁷ 5¹6¹
2.2 10 6.3 10 1.2 2.1 1.2 4.1 7.3 7.2 7.1 7.2

(1) (2) (3) V (4) (5) (6)

1⁹ 1³ 2⁹ 2³ 3⁹ 3³ 4⁹ 4³ 5⁹ 5³ 6⁹ 6³
5.2 6.2 5.2 8 7.2 7.3 5.2 6.2 9 3.1 9 1.1

(1) (2) (3) VI (4) (5) (6)

1¹¹ 1⁵2⁵ 2¹¹ 1⁵2⁵ 3¹¹ 3⁵ 4¹¹ 4⁵ 5¹¹ 5⁵ 6¹¹ 6⁵
1.1 1.3 4.2 1.3 5.1 6.3 1.3 1.1 6.3 5.1 6.1 5.1

Inverse Sides 1

(1) (2) (3) I (4) (5) (6)

1₁ 1¹ 2₁ 2¹ 3₁ 3¹ 4₁ 4¹ 5₁₆₁ 5¹⁶₁ 5₁₆₁ 5¹⁶₁

10 10 2.1 2.1 4.1 4.1 7.2 7.2

(1) (2) (3) II (4) (5) (6)

1₃ 1³ 2₃ 2³ 3₃ 3³ 4₃ 4³ 5₃ 5³ 6₃ 6³

6.2 8 7.3 7.3 6.2 6.2 3.1 3.1 1.1

(1) (2) (3) III (4) (5) (6)

1₅₂₅ 1⁵²₅ 1₅₂₅ 1⁵²₅ 3₅ 3⁵ 4₅ 4⁵ 5₅ 5⁵ 6₅ 6⁵

1.3 1.3 1.3 1.3 6.3 6.3 1.1 1.1 5.1 5.1 5.1 5.1

(1) (2) (3) IV (4) (5) (6)

1₇ 1⁷ 2₇ 2⁷ 3₇ 3⁷ 4₇ 4⁷ 5₇ 5⁷ 6₇ 6⁷

2.2 6.3 1.2 1.2 1.2 1.2 7.3 7.3 7.1 7.1

(1) (2) (3) V (4) (5) (6)

1₉ 1⁹ 2₉ 2⁹ 3₉ 3⁹ 4₉ 4⁹ 5₉ 5⁹ 6₉ 6⁹

5.2 5.2 7.2 7.2 5.2 5.2 9 9 9 9

(1) (2) (3) VI (4) (5) (6)

1₁₁ 1¹¹ 2₁₁ 2¹¹ 3₁₁ 3¹¹ 4₁₁ 4¹¹ 5₁₁ 5¹¹ 6₁₁ 6¹¹

1.1 4.2 5.1 5.1 1.3 1.3 6.3 6.3 6.1 6.1

Inverse Sides 2

(1) (2) (3) I (4) (5) (6)

1¹ 1₁ 2¹ 2₁ 3¹ 3₁ 4¹ 4₁ 5¹6¹ 5₁6₁ 5¹6¹ 5₁6₁

10 10 2.1 2.1 7.2 7.2

(1) (2) (3) II (4) (5) (6)

1³ 1₃ 2³ 2₃ 3³ 3₃ 4³ 4₃ 5³ 5₃ 6³ 6₃

6.2 8 7.3 7.3 3.1 1.1

(1) (2) (3) III (4) (5) (6)

1⁵2⁵ 1₅2₅ 1⁵2⁵ 1₅2₅ 3⁵ 3₅ 4⁵ 4₅ 5⁵ 5₅ 6⁵ 6₅

1.3 1.3 6.3 6.3 1.1 5.1 5.1

(1) (2) (3) IV (4) (5) (6)

1⁷ 1₇ 2⁷ 2₇ 3⁷ 3₇ 4⁷ 4₇ 5⁷ 5₇ 6⁷ 6₇

2.2 6.3 1.2 1.2 7.3 7.1

(1) (2) (3) V (4) (5) (6)

1⁹ 1₉ 2⁹ 2₉ 3⁹ 3₉ 4⁹ 4₉ 5⁹ 5₉ 6⁹ 6₉

5.2 5.2 7.2 7.2 9 9

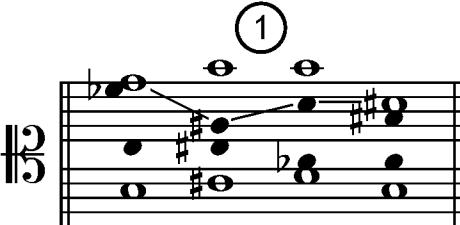
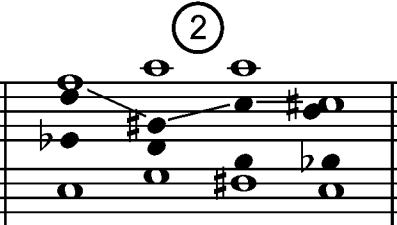
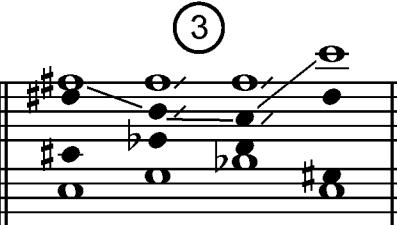
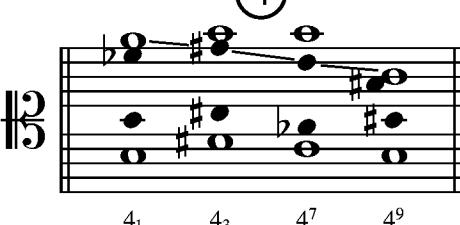
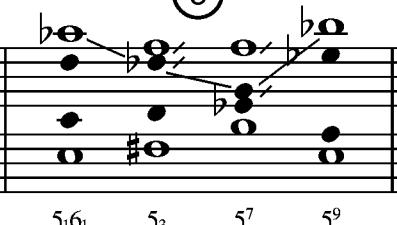
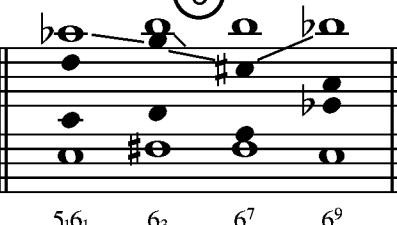
(1) (2) (3) VI (4) (5) (6)

1¹¹ 1₁₁ 2¹¹ 2₁₁ 3¹¹ 3₁₁ 4¹¹ 4₁₁ 5¹¹ 5₁₁ 6¹¹ 6₁₁

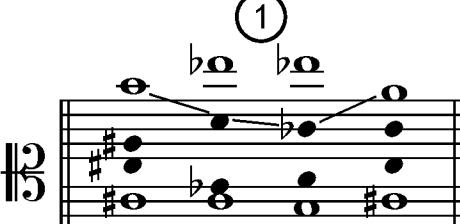
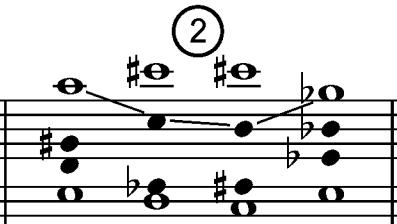
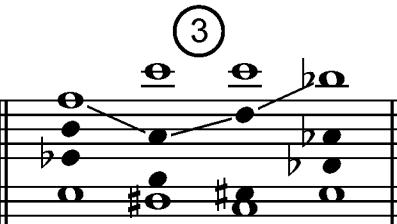
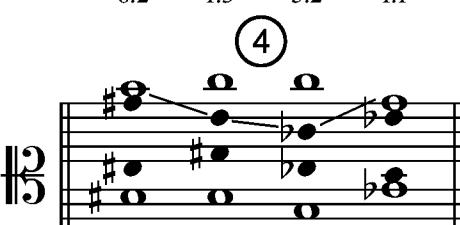
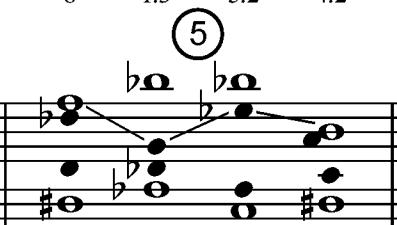
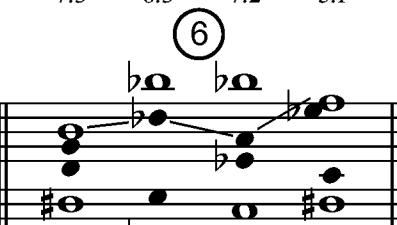
1.1 4.2 5.1 5.1 1.3 6.3 6.1

Interlinked Sides 1...

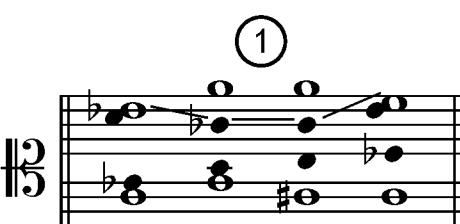
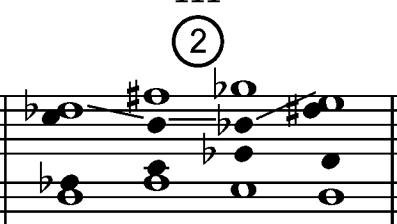
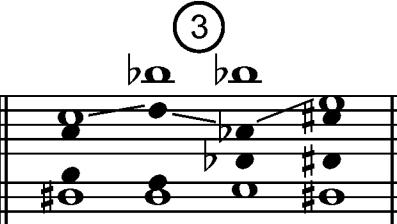
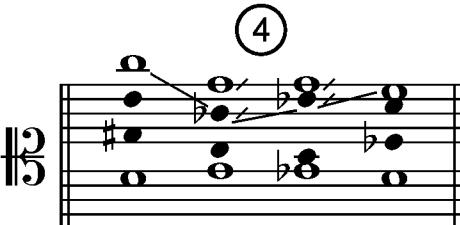
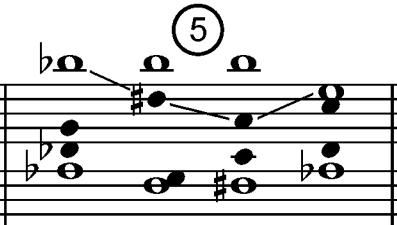
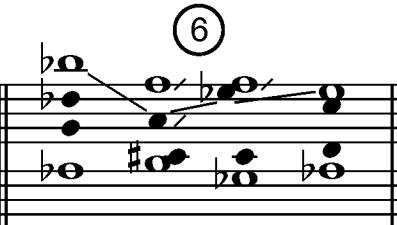
I

		
1 ₁ 1 ₃ 1 ⁷ 1 ⁹ 10 6.2 2.2 5.2	2 ₁ 2 ₃ 2 ⁷ 2 ⁹ 10 8 6.3 5.2	3 ₁ 3 ₃ 3 ⁷ 3 ⁹ 2.1 7.3 1.2 7.2
		
4 ₁ 4 ₃ 4 ⁷ 4 ⁹ 4.1 6.2 1.2 5.2	5 ₁ 5 ₃ 5 ⁷ 5 ⁹ 7.2 3.1 7.3 9	5 ₁ 6 ₃ 6 ⁷ 6 ⁹ 7.2 1.1 7.1 9

II

		
1 ₃ 1 ₅ 2 ₅ 1 ⁹ 1 ¹¹ 6.2 1.3 5.2 1.1	2 ₃ 1 ₅ 2 ₅ 2 ⁹ 2 ¹¹ 8 1.3 5.2 4.2	3 ₃ 3 ₅ 3 ⁹ 3 ¹¹ 7.3 6.3 7.2 5.1
		
4 ₃ 4 ₅ 4 ⁹ 4 ¹¹ 6.2 1.1 5.2 1.3	5 ₃ 5 ₅ 5 ⁹ 5 ¹¹ 3.1 5.1 9 6.3	6 ₃ 6 ₅ 6 ⁹ 6 ¹¹ 1.1 5.1 9 6.1

III

		
1 ₅ 2 ₅ 1 ₇ 1 ¹¹ 1 ¹ 1.3 2.2 1.1 10	1 ₅ 2 ₅ 2 ₇ 2 ¹¹ 2 ¹ 1.3 6.3 4.2 10	3 ₅ 3 ₇ 3 ¹¹ 3 ¹ 6.3 1.2 5.1 2.1
		
4 ₅ 4 ₇ 4 ¹¹ 4 ¹ 1.1 1.2 1.3 4.1	5 ₅ 5 ₇ 5 ¹¹ 5 ¹⁶ 5.1 7.3 6.3 7.2	6 ₅ 6 ₇ 6 ¹¹ 5 ¹⁶ 5.1 7.1 6.1 7.2

...Interlinked Sides 1

IV

(2)

Musical staff showing notes on five-line staff with a key signature of one sharp (F#). The staff begins with a bass clef, followed by a sharp sign, and ends with a double bar line and repeat sign.

1₇ 1₉ 1¹ 1³
2.2 5.2 10 6.2

Musical staff showing notes on five-line staff with a key signature of one sharp (F#).

2₇ 2₉ 2¹ 2³
6.3 5.2 10 8

Musical staff showing notes on five-line staff with a key signature of one sharp (F#).

3₇ 3₉ 3¹ 3³
1.2 7.2 2.1 7.3

(4)

Musical staff showing notes on five-line staff with a key signature of one sharp (F#).

4₇ 4₉ 4¹ 4³
1.2 5.2 4.1 6.2

(5)

Musical staff showing notes on five-line staff with a key signature of one sharp (F#).

5₇ 5₉ 5¹ 6¹ 5³
7.3 9 7.2 3.1

(6)

Musical staff showing notes on five-line staff with a key signature of one sharp (F#).

6₇ 6₉ 5¹ 6¹ 6³
7.1 9 7.2 1.1

V

(2)

Musical staff showing notes on five-line staff with a key signature of one sharp (F#).

1₉ 1₁₁ 1³ 1⁵ 2⁵
5.2 1.1 6.2 1.3

Musical staff showing notes on five-line staff with a key signature of one sharp (F#).

3₉ 3₁₁ 3³ 3⁵
7.2 5.1 7.3 6.3

(4)

Musical staff showing notes on five-line staff with a key signature of one sharp (F#).

4₉ 4₁₁ 4³ 4⁵
5.2 1.3 6.2 1.1

(5)

Musical staff showing notes on five-line staff with a key signature of one sharp (F#).

6₉ 6₁₁ 6³ 6⁵
9 6.1 1.1 5.1

VI

(2)

Musical staff showing notes on five-line staff with a key signature of one sharp (F#).

1₁₁ 1₁ 1⁵ 2⁵ 1⁷
1.1 10 1.3 2.2

Musical staff showing notes on five-line staff with a key signature of one sharp (F#).

3₁₁ 3₁ 3⁵ 3⁷
5.1 2.1 6.3 1.2

(4)

Musical staff showing notes on five-line staff with a key signature of one sharp (F#).

2₁₁ 2₁ 1⁵ 2⁵ 2⁷
4.2 10 1.3 6.3

Musical staff showing notes on five-line staff with a key signature of one sharp (F#).

3₁₁ 3₁ 3⁵ 3⁷
5.1 2.1 6.3 1.2

(5)

Musical staff showing notes on five-line staff with a key signature of one sharp (F#).

4₁₁ 4₁ 4⁵ 4⁷
1.3 4.1 1.1 1.2

Musical staff showing notes on five-line staff with a key signature of one sharp (F#).

5₁₁ 5₁₆₁ 5⁵ 5⁷
6.3 7.2 5.1 7.3

Musical staff showing notes on five-line staff with a key signature of one sharp (F#).

6₁₁ 5₁₆₁ 6⁵ 6⁷
6.1 7.2 5.1 7.1

3.10

Interlinked Sides 2...

I

 1 ₃ 1 ₁ 1 ⁹ 1 ⁷ 6.2 10 5.2 2.2	 2 ₃ 2 ₁ 2 ⁹ 2 ⁷ 8 10 5.2 6.3	 3 ₃ 3 ₁ 3 ⁹ 3 ⁷ 7.3 2.1 7.2 1.2
 4 ₃ 4 ₁ 4 ⁹ 4 ⁷ 6.2 4.1 5.2 1.2	 5 ₃ 5 ₁₆₁ 5 ⁹ 5 ⁷ 3.1 7.2 9 7.3	 6 ₃ 5 ₁₆₁ 6 ⁹ 6 ⁷ 1.1 7.2 9 7.1

II

 1 ₅₂₅ 1 ₃ 1 ¹¹ 1 ⁹ 1.3 6.2 1.1 5.2	 1 ₅₂₅ 2 ₃ 2 ¹¹ 2 ⁹ 1.3 8 4.2 5.2	 3 ₅ 3 ₃ 3 ¹¹ 3 ⁹ 6.3 7.3 5.1 7.2
 4 ₅ 4 ₃ 4 ¹¹ 4 ⁹ 1.1 6.2 1.3 5.2	 5 ₅ 5 ₃ 5 ¹¹ 5 ⁹ 5.1 3.1 6.3 9	 6 ₅ 6 ₃ 6 ¹¹ 6 ⁹ 5.1 1.1 6.1 9

III

 1 ₇ 1 ₅₂₅ 1 ¹ 1 ¹¹ 2.2 1.3 10 1.1	 2 ₇ 1 ₅₂₅ 2 ¹ 2 ¹¹ 6.3 1.3 10 4.2	 3 ₇ 3 ₅ 3 ¹ 3 ¹¹ 1.2 6.3 2.1 5.1
 4 ₇ 4 ₅ 4 ¹ 4 ¹¹ 1.2 1.1 4.1 1.3	 5 ₇ 5 ₅ 5 ¹⁶¹ 5 ¹¹ 7.3 5.1 7.2 6.3	 6 ₇ 6 ₅ 5 ¹⁶¹ 6 ¹¹ 7.1 5.1 7.2 6.1

...Interlinked Sides 2

IV

(2)

Musical staff showing six notes on five-line staff. The notes are connected by horizontal lines. The notes are: C, D, E, F, G, A. The staff has a key signature of one sharp (F#) and a common time signature.

1₉ 1₇ 1³ 1¹
5.2 2.2 6.2 10

Musical staff showing six notes on five-line staff. The notes are connected by horizontal lines. The notes are: C, D, E, F, G, A. The staff has a key signature of one sharp (F#) and a common time signature.

2₉ 2₇ 2³ 2¹
5.2 6.3 8 10

Musical staff showing six notes on five-line staff. The notes are connected by horizontal lines. The notes are: C, D, E, F, G, A. The staff has a key signature of one sharp (F#) and a common time signature.

3₉ 3₇ 3³ 3¹
7.2 1.2 7.3 2.1

(4)

Musical staff showing six notes on five-line staff. The notes are connected by horizontal lines. The notes are: C, D, E, F, G, A. The staff has a key signature of one sharp (F#) and a common time signature.

4₉ 4₇ 4³ 4¹
5.2 1.2 6.2 4.1

(5)

Musical staff showing six notes on five-line staff. The notes are connected by horizontal lines. The notes are: C, D, E, F, G, A. The staff has a key signature of one sharp (F#) and a common time signature.

5₉ 5₇ 5³ 5¹6¹
9 7.3 3.1 7.2

(6)

Musical staff showing six notes on five-line staff. The notes are connected by horizontal lines. The notes are: C, D, E, F, G, A. The staff has a key signature of one sharp (F#) and a common time signature.

6₉ 6₇ 6³ 5¹6¹
9 7.1 1.1 7.2

V

(2)

Musical staff showing six notes on five-line staff. The notes are connected by horizontal lines. The notes are: C, D, E, F, G, A. The staff has a key signature of one sharp (F#) and a common time signature.

1₁₁ 1₉ 1⁵2⁵ 1³
1.1 5.2 1.3 6.2

Musical staff showing six notes on five-line staff. The notes are connected by horizontal lines. The notes are: C, D, E, F, G, A. The staff has a key signature of one sharp (F#) and a common time signature.

2₁₁ 2₉ 1⁵2⁵ 2³
4.2 5.2 1.3 8

Musical staff showing six notes on five-line staff. The notes are connected by horizontal lines. The notes are: C, D, E, F, G, A. The staff has a key signature of one sharp (F#) and a common time signature.

3₁₁ 3₉ 3⁵ 3³
5.1 7.2 6.3 7.3

(4)

Musical staff showing six notes on five-line staff. The notes are connected by horizontal lines. The notes are: C, D, E, F, G, A. The staff has a key signature of one sharp (F#) and a common time signature.

4₁₁ 4₉ 4⁵ 4³
1.3 5.2 1.1 6.2

(5)

Musical staff showing six notes on five-line staff. The notes are connected by horizontal lines. The notes are: C, D, E, F, G, A. The staff has a key signature of one sharp (F#) and a common time signature.

5₁₁ 5₉ 5⁵ 5³
6.3 9 5.1 3.1

(6)

Musical staff showing six notes on five-line staff. The notes are connected by horizontal lines. The notes are: C, D, E, F, G, A. The staff has a key signature of one sharp (F#) and a common time signature.

6₁₁ 6₉ 6⁵ 6³
6.1 9 5.1 1.1

VI

(2)

Musical staff showing six notes on five-line staff. The notes are connected by horizontal lines. The notes are: C, D, E, F, G, A. The staff has a key signature of one sharp (F#) and a common time signature.

1₁ 1₁₁ 1⁷ 1⁵2⁵
10 1.1 2.2 1.3

Musical staff showing six notes on five-line staff. The notes are connected by horizontal lines. The notes are: C, D, E, F, G, A. The staff has a key signature of one sharp (F#) and a common time signature.

2₁ 2₁₁ 2⁷ 1⁵2⁵
10 4.2 6.3 1.3

Musical staff showing six notes on five-line staff. The notes are connected by horizontal lines. The notes are: C, D, E, F, G, A. The staff has a key signature of one sharp (F#) and a common time signature.

3₁ 3₁₁ 3⁷ 3⁵
2.1 5.1 1.2 6.3

(4)

Musical staff showing six notes on five-line staff. The notes are connected by horizontal lines. The notes are: C, D, E, F, G, A. The staff has a key signature of one sharp (F#) and a common time signature.

4₁ 4₁₁ 4⁷ 4⁵
4.1 1.3 1.2 1.1

(5)

Musical staff showing six notes on five-line staff. The notes are connected by horizontal lines. The notes are: C, D, E, F, G, A. The staff has a key signature of one sharp (F#) and a common time signature.

5₁₆ 5₁₁ 5⁷ 5⁵
7.2 6.3 7.3 5.1

(6)

Musical staff showing six notes on five-line staff. The notes are connected by horizontal lines. The notes are: C, D, E, F, G, A. The staff has a key signature of one sharp (F#) and a common time signature.

5₁₆ 6₁₁ 6⁷ 6⁵
7.2 6.1 7.1 5.1

3.12

Interlinked Sides Joined in Chains 1

(1)

(1)

1₁ 1₃ 1⁷ 1⁹ 1¹¹ 1₃ 1₅2₅ 1₇ 1¹¹ 1¹ 1³ 1₇ 1₉ 1₁₁ 1³ 1₅2₅ 1⁷ 1₁₁ 1₁

10 6.2 2.2 5.2 1.1 6.2 1.3 2.2 1.1 10 6.2 2.2 5.2 1.1 6.2 1.3 2.2 1.1 10

(2)

(2)

2₁ 2₃ 2⁷ 2⁹ 2¹¹ 2₃ 1₅2₅ 2₇ 2¹¹ 2¹ 2³ 2₇ 2₉ 2₁₁ 2³ 1⁵2₅ 2⁷ 2₁₁ 2₁

10 8 6.3 5.2 4.2 8 1.3 6.3 4.2 10 8 6.3 5.2 4.2 8 1.3 6.3 4.2 10

(3)

(3)

3₁ 3₃ 3⁷ 3⁹ 3¹¹ 3₃ 3₅ 3₇ 3¹¹ 3¹ 3³ 3₇ 3₉ 3₁₁ 3³ 3⁵ 3⁷ 3₁₁ 3₁

2.1 7.3 1.2 7.2 5.1 7.3 6.3 1.2 5.1 2.1 7.3 1.2 7.2 5.1 7.3 6.3 1.2 5.1 2.1

(4)

(4)

4₁ 4₃ 4⁷ 4⁹ 4¹¹ 4₃ 4₅ 4₇ 4¹¹ 4¹ 4³ 4₇ 4₉ 4₁₁ 4³ 4⁵ 4⁷ 4₁₁ 4₁

4.1 6.2 1.2 5.2 1.3 6.2 1.1 1.2 1.3 4.1 6.2 1.2 5.2 1.3 6.2 1.1 1.2 1.3 4.1

(5)

(5)

5₁6₁ 5₃ 5⁷ 5⁹ 5¹¹ 5₃ 5₅ 5₇ 5¹¹ 5¹6¹ 5³ 5₇ 5₉ 5₁₁ 5³ 5⁵ 5⁷ 5₁₁ 5₁6₁

7.2 3.1 7.3 9 6.3 3.1 5.1 7.3 6.3 7.2 3.1 7.3 9 6.3 3.1 5.1 7.3 6.3 7.2

(6)

(6)

5₁6₁ 6₃ 6⁷ 6⁹ 6¹¹ 6₃ 6₅ 6₇ 6¹¹ 5¹6¹ 6³ 6₇ 6₉ 6₁₁ 6³ 6⁵ 6⁷ 6₁₁ 5₁6₁

7.2 1.1 7.1 9 6.1 1.1 5.1 7.1 6.1 7.2 1.1 7.1 9 6.1 1.1 5.1 7.1 6.1 7.2

Interlinked Sides Joined in Chains 2

(1)

(1)

1^7 1^9 1_1 1_3 1_{525} 1^9 1^{11} 1^1 1_{525} 1_7 1_9 1^1 1^3 1^{525} 1_9 1_{11} 1_1 1^{525} 1^7
 2.2 5.2 10 6.2 1.3 5.2 1.1 10 1.3 2.2 5.2 10 6.2 1.3 5.2 1.1 10 1.3 2.2

(2)

(2)

2^7 2^9 2_1 2_3 1_{525} 2^9 2^{11} 2^1 1_{525} 2_7 2_9 2^1 2^3 1^{525} 2_9 2_{11} 2_1 1^{525} 2^7
 6.3 5.2 10 8 1.3 5.2 4.2 10 1.3 6.3 5.2 10 8 1.3 5.2 4.2 10 1.3 6.3

(3)

(3)

3^7 3^9 3_1 3_3 3_5 3^9 3^{11} 3^1 3_5 3_7 3_9 3^1 3^3 3^5 3_9 3_{11} 3_1 3^5 3^7
 1.2 7.2 2.1 7.3 6.3 7.2 5.1 2.1 6.3 1.2 7.2 2.1 7.3 6.3 7.2 5.1 2.1 6.3 1.2

(4)

(4)

4^7 4^9 4_1 4_3 4_5 4^9 4^{11} 4^1 4_5 4_7 4_9 4^1 4^3 4^5 4_9 4_{11} 4_1 4^5 4^7
 1.2 5.2 4.1 6.2 1.1 5.2 1.3 4.1 1.1 1.2 5.2 4.1 6.2 1.1 5.2 1.3 4.1 1.1 1.2

(5)

(5)

5^7 5^9 5_{161} 5_3 5_5 5^9 5^{11} 5^{161} 5_5 5_7 5_9 5^{161} 5^3 5^5 5_9 5_{11} 5_{161} 5^5 5^7
 7.3 9 7.2 3.1 5.1 9 6.3 7.2 5.1 7.3 9 7.2 3.1 5.1 9 6.3 7.2 5.1 7.3

(6)

(6)

6^7 6^9 5_{161} 6_3 6_5 6^9 6^{11} 5^{161} 6_5 6_7 6_9 5^{161} 6^3 6^5 6_9 6_{11} 5_{161} 6^5 6^7
 7.1 9 7.2 1.1 5.1 9 6.1 7.2 5.1 7.1 9 7.2 1.1 5.1 9 6.1 7.2 5.1 7.1

Horizontal Sides 1

(1)

10 → 1.1 ← 1¹ 1₃ → 6.2 ← 1³ 1₅2₅ → 1.3 ← 1⁵2₅ 1₇ → 2.2 ← 1⁷ 1₉ → 5.2 ← 1⁹ 1₁₁ → 1.1 ← 1¹¹

2₁ → 10 ← 2¹ 2₃ → 8 ← 2³ 1₅2₅ → 1.3 ← 1⁵2₅ 2₇ → 6.3 ← 2⁷ 2₉ → 5.2 ← 2⁹ 2₁₁ → 4.2 ← 2¹¹

3₁ → 2.1 ← 3¹ 3₃ → 7.3 ← 3³ 3₅ → 6.3 ← 3⁵ 3₇ → 1.2 ← 3⁷ 3₉ → 7.2 ← 3⁹ 3₁₁ → 5.1 ← 3¹¹

4₁ → 4.1 ← 4¹ 4₃ → 6.2 ← 4³ 4₅ → 1.1 ← 4⁵ 4₇ → 1.2 ← 4⁷ 4₉ → 5.2 ← 4⁹ 4₁₁ → 1.3 ← 4¹¹

5₁6₁ → 7.2 ← 5¹6¹ 5₃ → 3.1 ← 5³ 5₅ → 5.1 ← 5⁵ 5₇ → 7.3 ← 5⁷ 5₉ → 9 ← 5⁹ 5₁₁ → 6.3 ← 5¹¹

5₁6₁ → 7.2 ← 5¹6¹ 6₃ → 1.1 ← 6³ 6₅ → 5.1 ← 6⁵ 6₇ → 7.1 ← 6⁷ 6₉ → 9 ← 6⁹ 6₁₁ → 6.1 ← 6¹¹

Horizontal Sides 2

(1)

1¹ → 10 ← 1₁ 1³ → 6.2 ← 1₃ 1⁵2₅ → 1.3 ← 1₅2₅ 1⁷ → 2.2 ← 1₇ 1⁹ → 5.2 ← 1₉ 1¹¹ → 1.1 ← 1₁₁

2¹ → 10 ← 2₁ 2³ → 8 ← 2₃ 1⁵2₅ → 1.3 ← 1₅2₅ 2⁷ → 6.3 ← 2₇ 2⁹ → 5.2 ← 2₉ 2¹¹ → 4.2 ← 2₁₁

3¹ → 2.1 ← 3₁ 3³ → 7.3 ← 3₃ 3⁵ → 6.3 ← 3₅ 3⁷ → 1.2 ← 3₇ 3⁹ → 7.2 ← 3₉ 3¹¹ → 5.1 ← 3₁₁

4¹ → 4.1 ← 4₁ 4³ → 6.2 ← 4₃ 4⁵ → 1.1 ← 4₅ 4⁷ → 1.2 ← 4₇ 4⁹ → 5.2 ← 4₉ 4¹¹ → 1.3 ← 4₁₁

5¹6₁ → 7.2 ← 5₁6₁ 5³ → 3.1 ← 5₃ 5⁵ → 5.1 ← 5₅ 5⁷ → 7.3 ← 5₇ 5⁹ → 9 ← 5₉ 5¹¹ → 6.3 ← 5₁₁

5¹6₁ → 7.2 ← 5₁6₁ 6³ → 1.1 ← 6₃ 6⁵ → 5.1 ← 6₅ 6⁷ → 7.1 ← 6₇ 6⁹ → 9 ← 6₉ 6¹¹ → 6.1 ← 6₁₁

Horizontal Sides 3

(1)

1₁→ 10 ←1¹ 1³→ 6.2 ←1₃ 1₅2₅→ 1.3 ←1⁵2⁵ 1⁷→ 2.2 ←1₇ 1₉→ 5.2 ←1⁹ 1¹¹→ 1.1 ←1₁₁

2₁→ 10 ←2¹ 2³→ 8 ←2₃ 1₅2₅→ 1.3 ←1⁵2⁵ 2⁷→ 6.3 ←2₇ 2₉→ 5.2 ←2⁹ 2¹¹→ 4.2 ←2₁₁

3₁→ 2.1 ←3¹ 3³→ 7.3 ←3₃ 3₅→ 6.3 ←3⁵ 3⁷→ 1.2 ←3₇ 3₉→ 7.2 ←3⁹ 3¹¹→ 5.1 ←3₁₁

4₁→ 4.1 ←4¹ 4³→ 6.2 ←4₃ 4₅→ 1.1 ←4⁵ 4⁷→ 1.2 ←4₇ 4₉→ 5.2 ←4⁹ 4¹¹→ 1.3 ←4₁₁

5₁6₁→ 7.2 ←5¹6¹ 5³→ 3.1 ←5₃ 5₅→ 5.1 ←5⁵ 5⁷→ 7.3 ←5₇ 5₉→ 9 ←5⁹ 5¹¹→ 6.3 ←5₁₁

5₁6₁→ 7.2 ←5¹6¹ 6³→ 1.1 ←6₃ 6₅→ 5.1 ←6⁵ 6⁷→ 7.1 ←6₇ 6₉→ 9 ←6⁹ 6¹¹→ 6.1 ←6₁₁

Horizontal Sides 4

(1)

1₁→ 10 ←1₁ 1₃→ 6.2 ←1³ 1⁵2⁵→ 1.3 ←1₅2₅ 1₇→ 2.2 ←1⁷ 1₉→ 5.2 ←1₉ 1₁₁→ 1.1 ←1¹¹

2₁→ 10 ←2₁ 2₃→ 8 ←2³ 1⁵2⁵→ 1.3 ←1₅2₅ 2₇→ 6.3 ←2⁷ 2₉→ 5.2 ←2₉ 2₁₁→ 4.2 ←2¹¹

3₁→ 2.1 ←3₁ 3₃→ 7.3 ←3³ 3₅→ 6.3 ←3⁵ 3₇→ 1.2 ←3⁷ 3₉→ 7.2 ←3⁹ 3₁₁→ 5.1 ←3¹¹

4₁→ 4.1 ←4₁ 4₃→ 6.2 ←4³ 4₅→ 1.1 ←4₅ 4₇→ 1.2 ←4⁷ 4₉→ 5.2 ←4₉ 4₁₁→ 1.3 ←4¹¹

5₁6₁→ 7.2 ←5₁6₁ 5₃→ 3.1 ←5³ 5₅→ 5.1 ←5₅ 5₇→ 7.3 ←5⁷ 5₉→ 9 ←5₉ 5₁₁→ 6.3 ←5¹¹

5₁6₁→ 7.2 ←5₁6₁ 6₃→ 1.1 ←6³ 6₅→ 5.1 ←6₅ 6₇→ 7.1 ←6⁷ 6₉→ 9 ←6₉ 6₁₁→ 6.1 ←6¹¹

Horizontal Sides of Triangles 1

This section contains six staves of musical notation for triangles. Each staff begins with a downward-pointing triangle symbol (∇) and ends with an upward-pointing triangle symbol (Δ). The notes are represented by dots with stems. Numbered circles (1 through 6) are placed above certain notes to indicate specific performance techniques. The notes are labeled with various numbers such as 1, 10, 15, 25, 1.3, 19, 5.2, 13, 6.2, 17, 2.2, 11, 1.1, 21, 2.1, 35, 6.3, 39, 7.2, 33, 7.3, 37, 1.2, 41, 4.1, 45, 1.1, 49, 5.2, 43, 6.2, 47, 1.2, 56, 7.2, 55, 5.1, 59, 9, 53, 3.1, 57, 7.3, 511, 6.3, 65, 5.1, 69, 9, 63, 1.1, 67, 7.1, 611, 6.1, and 3.17.

Horizontal Sides of Triangles 2

This section contains six staves of musical notation for triangles, similar to the first section but with different note patterns. Each staff begins with a downward-pointing triangle symbol (∇) and ends with an upward-pointing triangle symbol (Δ). The notes are represented by dots with stems. Numbered circles (1 through 6) are placed above certain notes to indicate specific performance techniques. The notes are labeled with various numbers such as 1, 10, -1, 15, 25, 1.3, -15, 25, 19, -19, 13, 6.2, 17, -17, 2.2, 11, 1.1, 21, 10, -2, 15, 25, 1.3, -15, 25, 29, -29, 5.2, 23, 8, -23, 27, -27, 6.3, -27, 211, 4.2, -211, 31, 2.1, -31, 35, -35, 6.3, -35, 39, -39, 7.2, -39, 33, -33, 7.3, -33, 37, -37, 1.2, -37, 311, 5.1, -311, 41, -41, 45, -45, 1.1, -45, 49, -49, 5.2, -49, 43, -43, 6.2, -43, 47, -47, 1.2, -47, 411, 1.3, -411, 56, 7.2, -56, 55, -55, 5.1, -55, 59, -59, 9, -59, 53, -53, 3.1, -53, 57, -57, 7.3, -57, 511, 6.3, -511, 65, 5.1, -65, 69, -69, 9, -69, 63, -63, 1.1, -63, 67, -67, 7.1, -67, 611, 6.1, -611, and 3.17.

Horizontal Sides of Downward Triangles (∇)

(1)

(2)

(3)

(4)

(5)

(6)

Horizontal Sides of Upward Triangles (Δ)

(1)

(2)

(3)

(4)

(5)

(6)

3.18

Horizontal Opposite Sides 1

1 2 3 I 4 5 6

$1_1 \rightarrow 10 \quad \leftarrow 1^1 \quad 2_1 \rightarrow 10 \quad \leftarrow 2^1 \quad 3_1 \rightarrow 2.1 \quad \leftarrow 3^1 \quad 4_1 \rightarrow 4.1 \quad \leftarrow 4^1 \quad 5_{1,6} \rightarrow 7.2 \quad \leftarrow 5^1 6^1 \quad 5_{1,6} \rightarrow 7.2 \quad \leftarrow 5^1 6^1$

$1_7 \rightarrow 2.2 \quad \leftarrow 1^7 \quad 2_7 \rightarrow 6.3 \quad \leftarrow 2^7 \quad 3_7 \rightarrow 1.2 \quad \leftarrow 3^7 \quad 4_7 \rightarrow 1.2 \quad \leftarrow 4^7 \quad 5_7 \rightarrow 7.3 \quad \leftarrow 5^7 \quad 6_7 \rightarrow 7.1 \quad \leftarrow 6^7$

1 2 3 II 4 5 6

$1_3 \rightarrow 6.2 \quad \leftarrow 1^3 \quad 2_3 \rightarrow 8 \quad \leftarrow 2^3 \quad 3_3 \rightarrow 7.3 \quad \leftarrow 3^3 \quad 4_3 \rightarrow 6.2 \quad \leftarrow 4^3 \quad 5_3 \rightarrow 3.1 \quad \leftarrow 5^3 \quad 6_3 \rightarrow 1.1 \quad \leftarrow 6^3$

$1_9 \rightarrow 5.2 \quad \leftarrow 1^9 \quad 2_9 \rightarrow 5.2 \quad \leftarrow 2^9 \quad 3_9 \rightarrow 7.2 \quad \leftarrow 3^9 \quad 4_9 \rightarrow 5.2 \quad \leftarrow 4^9 \quad 5_9 \rightarrow 9 \quad \leftarrow 5^9 \quad 6_9 \rightarrow 9 \quad \leftarrow 6^9$

1 2 3 III 4 5 6

$1_{15,25} \rightarrow 1.3 \quad \leftarrow 1^5 2^5 \quad 1_{15,25} \rightarrow 1.3 \quad \leftarrow 1^5 2^5 \quad 3_5 \rightarrow 6.3 \quad \leftarrow 3^5 \quad 4_5 \rightarrow 1.1 \quad \leftarrow 4^5 \quad 5_5 \rightarrow 5.1 \quad \leftarrow 5^5 \quad 6_5 \rightarrow 5.1 \quad \leftarrow 6^5$

$1_{11} \rightarrow 1.1 \quad \leftarrow 1^{11} \quad 2_{11} \rightarrow 4.2 \quad \leftarrow 2^{11} \quad 3_{11} \rightarrow 5.1 \quad \leftarrow 3^{11} \quad 4_{11} \rightarrow 1.3 \quad \leftarrow 4^{11} \quad 5_{11} \rightarrow 6.3 \quad \leftarrow 5^{11} \quad 6_{11} \rightarrow 6.1 \quad \leftarrow 6^{11}$

1 2 3 IV 4 5 6

$1_7 \rightarrow 10 \quad \leftarrow 1^7 \quad 2_7 \rightarrow 10 \quad \leftarrow 2^7 \quad 3_7 \rightarrow 2.1 \quad \leftarrow 3^7 \quad 4_7 \rightarrow 4.1 \quad \leftarrow 4^7 \quad 5_7 \rightarrow 7.2 \quad \leftarrow 5^7 \quad 6_7 \rightarrow 7.2 \quad \leftarrow 6^7$

$1_1 \rightarrow 2.2 \quad \leftarrow 1^1 \quad 2_1 \rightarrow 6.3 \quad \leftarrow 2^1 \quad 3_1 \rightarrow 1.2 \quad \leftarrow 3^1 \quad 4_1 \rightarrow 1.2 \quad \leftarrow 4^1 \quad 5_{1,6} \rightarrow 7.3 \quad \leftarrow 5^1 6^1 \quad 5_{1,6} \rightarrow 7.1 \quad \leftarrow 5^1 6^1$

1 2 3 V 4 5 6

$1_9 \rightarrow 6.2 \quad \leftarrow 1^9 \quad 2_9 \rightarrow 8 \quad \leftarrow 2^9 \quad 3_9 \rightarrow 7.3 \quad \leftarrow 3^9 \quad 4_9 \rightarrow 6.2 \quad \leftarrow 4^9 \quad 5_9 \rightarrow 3.1 \quad \leftarrow 5^9 \quad 6_9 \rightarrow 1.1 \quad \leftarrow 6^9$

$1_3 \rightarrow 5.2 \quad \leftarrow 1^3 \quad 2_3 \rightarrow 5.2 \quad \leftarrow 2^3 \quad 3_3 \rightarrow 7.2 \quad \leftarrow 3^3 \quad 4_3 \rightarrow 5.2 \quad \leftarrow 4^3 \quad 5_3 \rightarrow 9 \quad \leftarrow 5^3 \quad 6_3 \rightarrow 9 \quad \leftarrow 6^3$

1 2 3 VI 4 5 6

$1_{11} \rightarrow 1.3 \quad \leftarrow 1^{11} \quad 2_{11} \rightarrow 1.3 \quad \leftarrow 2^{11} \quad 3_{11} \rightarrow 6.3 \quad \leftarrow 3^{11} \quad 4_{11} \rightarrow 1.1 \quad \leftarrow 4^{11} \quad 5_{11} \rightarrow 5.1 \quad \leftarrow 5^{11} \quad 6_{11} \rightarrow 5.1 \quad \leftarrow 6^{11}$

$1_{15,25} \rightarrow 1.1 \quad \leftarrow 1^5 2^5 \quad 1_{15,25} \rightarrow 4.2 \quad \leftarrow 1^5 2^5 \quad 3_5 \rightarrow 5.1 \quad \leftarrow 3^5 \quad 4_5 \rightarrow 1.3 \quad \leftarrow 4^5 \quad 5_5 \rightarrow 6.3 \quad \leftarrow 5^5 \quad 6_5 \rightarrow 6.1 \quad \leftarrow 6^5$

Horizontal Opposite Sides 2

(1) (2) (3) I (4) (5) (6)

1¹→ 10 ←1₁ 2¹→ 10 ←2₁ 3¹→ 2.1 ←3₁ 4¹→ 4.1 ←4₁ 5¹6¹→ 7.2 ←5₁6₁ 5¹6¹→ 7.2 ←5₁6₁

1⁷→ 2.2 ←1₇ 2⁷→ 6.3 ←2₇ 3⁷→ 1.2 ←3₇ 4⁷→ 1.2 ←4₇ 5⁷→ 7.3 ←5₇ 6⁷→ 7.1 ←6₇

1³→ 6.2 ←1₃ 2³→ 8 ←2₃ 3³→ 7.3 ←3₃ 4³→ 6.2 ←4₃ 5³→ 3.1 ←5₃ 6³→ 1.1 ←6₃

1⁹→ 5.2 ←1₉ 2⁹→ 5.2 ←2₉ 3⁹→ 7.2 ←3₉ 4⁹→ 5.2 ←4₉ 5⁹→ 9 ←5₉ 6⁹→ 9 ←6₉

1⁵2⁵→ 1.3 ←1₅2₅ 1⁵2⁵→ 1.3 ←1₅2₅ 3⁵→ 6.3 ←3₅ 4⁵→ 1.1 ←4₅ 5⁵→ 5.1 ←5₅ 6⁵→ 5.1 ←6₅

1¹¹→ 1.1 ←1₁₁ 2¹¹→ 4.2 ←2₁₁ 3¹¹→ 5.1 ←3₁₁ 4¹¹→ 1.3 ←4₁₁ 5¹¹→ 6.3 ←5₁₁ 6¹¹→ 6.1 ←6₁₁

1⁷→ 10 ←1₇ 2⁷→ 10 ←2₇ 3⁷→ 2.1 ←3₇ 4⁷→ 4.1 ←4₇ 5⁷→ 7.2 ←5₇ 6⁷→ 7.2 ←6₇

1¹→ 2.2 ←1₁ 2¹→ 6.3 ←2₁ 3¹→ 1.2 ←3₁ 4¹→ 1.2 ←4₁ 5¹6¹→ 7.3 ←5₁6₁ 5¹6¹→ 7.1 ←5₁6₁

1⁹→ 6.2 ←1₉ 2⁹→ 8 ←2₉ 3⁹→ 7.3 ←3₉ 4⁹→ 6.2 ←4₉ 5⁹→ 3.1 ←5₉ 6⁹→ 1.1 ←6₉

1³→ 5.2 ←1₃ 2³→ 5.2 ←2₃ 3³→ 7.2 ←3₃ 4³→ 5.2 ←4₃ 5³→ 9 ←5₃ 6³→ 9 ←6₃

1¹¹→ 1.3 ←1₁₁ 2¹¹→ 4.2 ←2₁₁ 3¹¹→ 6.3 ←3₁₁ 4¹¹→ 1.1 ←4₁₁ 5¹¹→ 5.1 ←5₁₁ 6¹¹→ 5.1 ←6₁₁

1⁷→ 10 ←1₇ 2⁷→ 10 ←2₇ 3⁷→ 2.1 ←3₇ 4⁷→ 4.1 ←4₇ 5⁷→ 7.2 ←5₇ 6⁷→ 7.2 ←6₇

1¹→ 2.2 ←1₁ 2¹→ 6.3 ←2₁ 3¹→ 1.2 ←3₁ 4¹→ 1.2 ←4₁ 5¹6¹→ 7.3 ←5₁6₁ 5¹6¹→ 7.1 ←5₁6₁

1⁹→ 6.2 ←1₉ 2⁹→ 8 ←2₉ 3⁹→ 7.3 ←3₉ 4⁹→ 6.2 ←4₉ 5⁹→ 3.1 ←5₉ 6⁹→ 1.1 ←6₉

1³→ 5.2 ←1₃ 2³→ 5.2 ←2₃ 3³→ 7.2 ←3₃ 4³→ 5.2 ←4₃ 5³→ 9 ←5₃ 6³→ 9 ←6₃

1¹¹→ 1.3 ←1₁₁ 2¹¹→ 1.3 ←2₁₁ 3¹¹→ 6.3 ←3₁₁ 4¹¹→ 1.1 ←4₁₁ 5¹¹→ 5.1 ←5₁₁ 6¹¹→ 5.1 ←6₁₁

1⁵2⁵→ 1.1 ←1₅2₅ 1⁵2⁵→ 4.2 ←1₅2₅ 3⁵→ 5.1 ←3₅ 4⁵→ 1.3 ←4₅ 5⁵→ 6.3 ←5₅ 6⁵→ 6.1 ←6₅

Angles...

(1)

Musical score for section (1) featuring two staves. The first staff uses a key signature of one flat, and the second staff uses a key signature of one sharp. The music consists of eighth-note patterns and rests.

Below the score are the following note values and rests:

$1_1 \setminus 1_5 2_5$	$1^5 2^5 \setminus 1^1$	$1_3 \setminus 1_7$	$1^7 \setminus 1^3$	$1_5 2_5 \setminus 1_9$	$1^9 \setminus 1^5 2^5$	$1_7 \setminus 1_{11}$	$1^{11} \setminus 1^7$	$1_9 \setminus 1_1$	$1^1 \setminus 1^9$	$1_{11} \setminus 1_3$	$1^3 \setminus 1^{11}$
$10 \setminus 1.3$	$1.3 \setminus 10$	$6.2 \setminus 2.2$	$2.2 \setminus 6.2$	$1.3 \setminus 5.2$	$5.2 \setminus 1.3$	$2.2 \setminus 1.1$	$1.1 \setminus 2.2$	$5.2 \setminus 10$	$10 \setminus 5.2$	$1.1 \setminus 6.2$	$6.2 \setminus 1.1$

(2)

Musical score for section (2) featuring two staves. The first staff uses a key signature of one flat, and the second staff uses a key signature of one sharp. The music consists of eighth-note patterns and rests.

Below the score are the following note values and rests:

$2_1 \setminus 1_5 2_5$	$1^5 2^5 \setminus 2^1$	$2_3 \setminus 2_7$	$2^7 \setminus 2^3$	$1_5 2_5 \setminus 2_9$	$2^9 \setminus 1^5 2^5$	$2_7 \setminus 2_{11}$	$2^{11} \setminus 2^7$	$2_9 \setminus 2_1$	$2^1 \setminus 2^9$	$2_{11} \setminus 2_3$	$2^3 \setminus 2^{11}$
$10 \setminus 1.3$	$1.3 \setminus 10$	$8 \setminus 6.3$	$6.3 \setminus 8$	$1.3 \setminus 5.2$	$5.2 \setminus 1.3$	$6.3 \setminus 4.2$	$4.2 \setminus 6.3$	$5.2 \setminus 10$	$10 \setminus 5.2$	$4.2 \setminus 8$	$8 \setminus 4.2$

(3)

Musical score for section (3) featuring two staves. The first staff uses a key signature of one flat, and the second staff uses a key signature of one sharp. The music consists of eighth-note patterns and rests.

Below the score are the following note values and rests:

$3_1 \setminus 3_5$	$3^5 \setminus 3^1$	$3_3 \setminus 3_7$	$3^7 \setminus 3^3$	$3_5 \setminus 3_9$	$3^9 \setminus 3^5$	$3_7 \setminus 3_{11}$	$3^{11} \setminus 3^7$	$3_9 \setminus 3_1$	$3^1 \setminus 3^9$	$3_{11} \setminus 3_3$	$3^3 \setminus 3^{11}$
$2.1 \setminus 6.3$	$6.3 \setminus 2.1$	$7.3 \setminus 1.2$	$1.2 \setminus 7.3$	$6.3 \setminus 7.2$	$7.2 \setminus 6.3$	$1.2 \setminus 5.1$	$5.1 \setminus 1.2$	$7.2 \setminus 2.1$	$2.1 \setminus 7.2$	$5.1 \setminus 7.3$	$7.3 \setminus 5.1$

...Angles

(4)

Musical score for section 4, featuring 12 measures of music on a staff with a key signature of one sharp. The score consists of two systems of six measures each. Measures 1-6 and 7-12 are separated by a vertical dashed bar. The music includes various note heads, stems, and rests. Below the staff, a series of numbered chords are listed:

4₁\4₅ 4⁵\4¹ 4₃\4₇ 4⁷\4³ 4₅\4₉ 4⁹\4⁵ 4₇\4₁₁ 4¹¹\4⁷ 4₉\4₁ 4¹\4⁹ 4₁₁\4₃ 4³\4¹¹
 4.1\1.1 1.1\4.1 6.2\1.2 1.2\6.2 1.1\5.2 5.2\1.1 1.2\1.3 1.3\1.2 5.2\4.1 4.1\5.2 1.3\6.2 6.2\1.3

(5)

Musical score for section 5, featuring 12 measures of music on a staff with a key signature of one sharp. The score consists of two systems of six measures each. Measures 1-6 and 7-12 are separated by a vertical dashed bar. The music includes various note heads, stems, and rests. Below the staff, a series of numbered chords are listed:

5₁6₁\5₅ 5⁵\5¹6¹ 5₃\5₇ 5⁷\5³ 5₅\5₉ 5⁹\5⁵ 5₇\5₁₁ 5¹¹\5⁷ 5₉\5₁6₁ 5¹6¹\5⁹ 5₁₁\5₃ 5³\5¹¹
 7.2\5.1 5.1\7.2 3.1\7.3 7.3\3.1 5.1\9 9\5.1 7.3\6.3 6.3\7.3 9\7.2 7.2\9 6.3\3.1 3.1\6.3

(6)

Musical score for section 6, featuring 12 measures of music on a staff with a key signature of one sharp. The score consists of two systems of six measures each. Measures 1-6 and 7-12 are separated by a vertical dashed bar. The music includes various note heads, stems, and rests. Below the staff, a series of numbered chords are listed:

5₁6₁\6₅ 6⁵\5¹6¹ 6₃\6₇ 6⁷\6³ 6₅\6₉ 6⁹\6⁵ 6₇\6₁₁ 6¹¹\6⁷ 6₉\5₁6₁ 5¹6¹\6⁹ 6₁₁\6₃ 6³\6¹¹
 7.2\5.1 5.1\7.2 1.1\7.1 7.1\1.1 5.1\9 9\5.1 7.1\6.1 6.1\7.1 9\7.2 7.2\9 6.1\1.1 1.1\6.1

Crosses...

(1)

1₁\1₃ 1³\1¹ 1₃\1₅2₅ 1⁵2⁵\1³ 1₅2₅\1₇ 1⁷\1⁵2⁵ 1₇\1₉ 1⁹\1⁷ 1₉\1₁₁ 1¹¹\1⁹ 1₁₁\1₁ 1¹\1¹¹
 10\6.2 6.2\10 6.2\1.3 1.3\6.2 1.3\2.2 2.2\1.3 2.2\5.2 5.2\2.2 5.2\1.1 1.1\5.2 1.1\10 10\1.1

(2)

2₁\2₃ 2³\2¹ 2₃\1₅2₅ 1⁵2⁵\2³ 1₅2₅\2₇ 2⁷\1⁵2⁵ 2₇\2₉ 2⁹\2⁷ 2₉\2₁₁ 2¹¹\2⁹ 2₁₁\2₁ 2¹\2¹¹
 10\8 8\10 8\1.3 1.3\8 1.3\6.3 6.3\1.3 6.3\5.2 5.2\6.3 5.2\4.2 4.2\5.2 4.2\10 10\4.2

(3)

3₁\3₃ 3³\3¹ 3₃\3₅ 3⁵\3³ 3₅\3₇ 3⁷\3⁵ 3₇\3₉ 3⁹\3⁷ 3₉\3₁₁ 3¹¹\3⁹ 3₁₁\3₁ 3¹\3¹¹
 2.1\7.3 7.3\2.1 7.3\6.3 6.3\7.3 6.3\1.2 1.2\6.3 1.2\7.2 7.2\1.2 7.2\5.1 5.1\7.2 5.1\2.1 2.1\5.1

...Crosses

(4)

2 3 C||

4₁\4₃ 4³\4¹ 4₃\4₅ 4⁵\4³ 4₅\4₇ 4⁷\4⁵ 4₇\4₉ 4⁹\4⁷ 4₉\4₁₁ 4¹¹\4⁹ 4₁₁\4₁ 4¹\4¹¹
4.1\6.2 6.2\4.1 6.2\1.1 1.1\6.2 1.1\1.2 1.2\1.1 1.2\5.2 5.2\1.2 5.2\1.3 1.3\5.2 1.3\4.1 4.1\1.3

(5)

2 3 C||

5₁6₁\5₃ 5³\5¹6¹ 5₃\5₅ 5⁵\5³ 5₅\5₇ 5⁷\5⁵ 5₇\5₉ 5⁹\5⁷ 5₉\5₁₁ 5¹¹\5⁹ 5₁₁\5₁6₁ 5¹6¹\5¹¹
7.2\3.1 3.1\7.2 3.1\5.1 5.1\3.1 5.1\7.3 7.3\5.1 7.3\9 9\7.3 9\6.3 6.3\9 6.3\7.2 7.2\6.3

(6)

2 3 C||

5₁6₁\6₃ 6³\5¹6¹ 6₃\6₅ 6⁵\6³ 6₅\6₇ 6⁷\6⁵ 6₇\6₉ 6⁹\6⁷ 6₉\6₁₁ 6¹¹\6⁹ 6₁₁\5₁6₁ 5¹6¹\6¹¹
7.2\1.1 1.1\7.2 1.1\5.1 5.1\1.1 5.1\7.1 7.1\5.1 7.1\9 9\7.1 9\6.1 6.1\9 6.1\7.2 7.2\6.1

Horizontal Angles 1

(1)

1₁\1₅2₅ → 10\1.3 ←1¹\1⁵2⁵ 1₃\1₇ → 6.2\2.2 ←1³\1⁷ 1₅2₅\1₉ → 1.3\5.2 ←1⁵2⁵\1⁹

1₇\1₁₁ → 2.2\1.1 ←1⁷\1¹¹ 1₉\1₁ → 5.2\10 ←1⁹\1¹ 1₁₁\1₃ → 1.1\6.2 ←1¹¹\1³

(2)

2₁\1₅2₅ → 10\1.3 ←2¹\1⁵2⁵ 2₃\2₇ → 8\6.3 ←2³\2⁷ 1₅2₅\2₉ → 1.3\5.2 ←1⁵2⁵\2⁹

2₇\2₁₁ → 6.3\4.2 ←2⁷\2¹¹ 2₉\2₁ → 5.2\10 ←2⁹\2¹ 2₁₁\2₃ → 4.2\8 ←2¹¹\2³

(3)

3₁\3₅ → 2.1\6.3 ←3¹\3⁵ 3₃\3₇ → 7.3\1.2 ←3³\3⁷ 3₅\3₉ → 6.3\7.2 ←3⁵\3⁹

3₇\3₁₁ → 1.2\5.1 ←3⁷\3¹¹ 3₉\3₁ → 7.2\2.1 ←3⁹\3¹ 3₁₁\3₃ → 5.1\7.3 ←3¹¹\3³

(4)

4₁\4₅ → 4.1\1.1 ←4¹\4⁵ 4₃\4₇ → 6.2\1.2 ←4³\4⁷ 4₅\4₉ → 1.1\5.2 ←4⁵\4⁹

4₇\4₁₁ → 1.2\1.3 ←4⁷\4¹¹ 4₉\4₁ → 5.2\4.1 ←4⁹\4¹ 4₁₁\4₃ → 1.3\6.2 ←4¹¹\4³

(5)

5₆\5₅ → 7.2\5.1 ←5⁶\5⁵ 5₃\5₇ → 3.1\7.3 ←5³\5⁷ 5₅\5₉ → 5.1\9 ←5⁵\5⁹

5₇\5₁₁ → 7.3\6.3 ←5⁷\5¹¹ 5₉\5₁₆ → 9\7.2 ←5⁹\5⁶ 5₁₁\5₃ → 6.3\3.1 ←5¹¹\5³

(6)

5₁₆\6₅ → 7.2\5.1 ←5⁶\6⁵ 6₃\6₇ → 1.1\7.1 ←6³\6⁷ 6₅\6₉ → 5.1\9 ←6⁵\6⁹

6₇\6₁₁ → 7.1\6.1 ←6⁷\6¹¹ 6₉\5₆ → 9\7.2 ←6⁹\5⁶ 6₁₁\6₃ → 6.1\1.1 ←6¹¹\6³

* ♪ = corner point.

Horizontal Angles 2

(1)

Musical staff showing six measures of music. The notes are represented by dots on a five-line staff. Below each measure is a musical notation with a vertical bar and two numbers separated by a slash, indicating a specific pitch or rhythm. Measures include: $1^52^5\backslash 1^1 \rightarrow$, $1.3\backslash 10$, $\leftarrow 1_52_5\backslash 1_1$, $1^7\backslash 1^3 \rightarrow$, $2.2\backslash 6.2$, $\leftarrow 1_7\backslash 1_3$, $1^9\backslash 1^52^5 \rightarrow$, $5.2\backslash 1.3$, $\leftarrow 1_9\backslash 1_52_5$; $1^{11}\backslash 1^7 \rightarrow$, $1.1\backslash 2.2$, $\leftarrow 1_{11}\backslash 1_7$, $1^1\backslash 1^9 \rightarrow$, $10\backslash 5.2$, $\leftarrow 1_1\backslash 1_9$, $1^3\backslash 1^{11} \rightarrow$, $6.2\backslash 1.1$, $\leftarrow 1_3\backslash 1_{11}$.

(2)

Musical staff showing six measures of music. The notes are represented by dots on a five-line staff. Measures include: $1^52^5\backslash 2^1 \rightarrow$, $1.3\backslash 10$, $\leftarrow 1_52_5\backslash 2_1$, $2^7\backslash 2^3 \rightarrow$, $6.3\backslash 8$, $\leftarrow 2_7\backslash 2_3$, $2^9\backslash 1^52^5 \rightarrow$, $5.2\backslash 1.3$, $\leftarrow 2_9\backslash 1_52_5$; $2^{11}\backslash 2^7 \rightarrow$, $4.2\backslash 6.3$, $\leftarrow 2_{11}\backslash 2_7$, $2^1\backslash 2^9 \rightarrow$, $10\backslash 5.2$, $\leftarrow 2_1\backslash 2_9$, $2^3\backslash 2^{11} \rightarrow$, $8\backslash 4.2$, $\leftarrow 2_3\backslash 2_{11}$.

(3)

Musical staff showing six measures of music. The notes are represented by dots on a five-line staff. Measures include: $3^5\backslash 3^1 \rightarrow$, $6.3\backslash 2.1$, $\leftarrow 3_5\backslash 3_1$, $3^7\backslash 3^3 \rightarrow$, $1.2\backslash 7.3$, $\leftarrow 3_7\backslash 3_3$, $3^9\backslash 3^5 \rightarrow$, $7.2\backslash 6.3$, $\leftarrow 3_9\backslash 3_5$; $3^{11}\backslash 3^7 \rightarrow$, $5.1\backslash 1.2$, $\leftarrow 3_{11}\backslash 3_7$, $3^1\backslash 3^9 \rightarrow$, $2.1\backslash 7.2$, $\leftarrow 3_1\backslash 3_9$, $3^3\backslash 3^{11} \rightarrow$, $7.3\backslash 5.1$, $\leftarrow 3_3\backslash 3_{11}$.

(4)

Musical staff showing six measures of music. The notes are represented by dots on a five-line staff. Measures include: $4^5\backslash 4^1 \rightarrow$, $1.1\backslash 4.1$, $\leftarrow 4_5\backslash 4_1$, $4^7\backslash 4^3 \rightarrow$, $1.2\backslash 6.2$, $\leftarrow 4_7\backslash 4_3$, $4^9\backslash 4^5 \rightarrow$, $5.2\backslash 1.1$, $\leftarrow 4_9\backslash 4_5$; $4^{11}\backslash 4^7 \rightarrow$, $1.3\backslash 1.2$, $\leftarrow 4_{11}\backslash 4_7$, $4^1\backslash 4^9 \rightarrow$, $4.1\backslash 5.2$, $\leftarrow 4_1\backslash 4_9$, $4^3\backslash 4^{11} \rightarrow$, $6.2\backslash 1.3$, $\leftarrow 4_3\backslash 4_{11}$.

(5)

Musical staff showing six measures of music. The notes are represented by dots on a five-line staff. Measures include: $5^5\backslash 5^6\backslash 1 \rightarrow$, $5.1\backslash 7.2$, $\leftarrow 5_5\backslash 5_6\backslash 1$, $5^7\backslash 5^3 \rightarrow$, $7.3\backslash 3.1$, $\leftarrow 5_7\backslash 5_3$, $5^9\backslash 5^5 \rightarrow$, $9\backslash 5.1$, $\leftarrow 5_9\backslash 5_5$; $5^{11}\backslash 5^7 \rightarrow$, $6.3\backslash 7.3$, $\leftarrow 5_{11}\backslash 5_7$, $5^1\backslash 5^9 \rightarrow$, $7.2\backslash 9$, $\leftarrow 5_1\backslash 5_9$, $5^3\backslash 5^{11} \rightarrow$, $3.1\backslash 6.3$, $\leftarrow 5_3\backslash 5_{11}$.

(6)

Musical staff showing six measures of music. The notes are represented by dots on a five-line staff. Measures include: $6^5\backslash 5^6\backslash 1 \rightarrow$, $5.1\backslash 7.2$, $\leftarrow 6_5\backslash 5_6\backslash 1$, $6^7\backslash 6^3 \rightarrow$, $7.1\backslash 1.1$, $\leftarrow 6_7\backslash 6_3$, $6^9\backslash 6^5 \rightarrow$, $9\backslash 5.1$, $\leftarrow 6_9\backslash 6_5$; $6^{11}\backslash 6^7 \rightarrow$, $6.1\backslash 7.1$, $\leftarrow 6_{11}\backslash 6_7$, $5^1\backslash 6^9 \rightarrow$, $7.2\backslash 9$, $\leftarrow 5_1\backslash 6_9$, $6^3\backslash 6^{11} \rightarrow$, $1.1\backslash 6.1$, $\leftarrow 6_3\backslash 6_{11}$.

* = corner point.

Horizontal Crosses 1

(1)

Musical staff showing horizontal crosses. The staff has two measures of music. The first measure consists of notes on the 1st, 3rd, 5th, and 7th steps of the scale. The second measure consists of notes on the 10th, 12th, and 14th steps. The notes are connected by horizontal lines between them.

1₁\1₃→ 10\6.2 ←1¹\1³ 1₃\1₅2₅→ 6.2\1.3 ←1³\1⁵2⁵ 1₅2₅\1₇→ 1.3\2.2 ←1⁵2⁵\1⁷

1₇\1₉→ 2.2\5.2 ←1⁷\1⁹ 1₉\1₁₁→ 5.2\1.1 ←1⁹\1¹¹ 1₁₁\1₁→ 1.1\10 ←1¹¹\1¹

(2)

Musical staff showing horizontal crosses. The staff has two measures of music. The first measure consists of notes on the 2nd, 4th, and 6th steps of the scale. The second measure consists of notes on the 10th, 12th, and 14th steps. The notes are connected by horizontal lines between them.

2₁\2₃→ 10\8 ←2¹\2³ 2₃\1₅2₅→ 8\1.3 ←2³\1⁵2⁵ 1₅2₅\2₇→ 1.3\6.3 ←1⁵2⁵\2⁷

2₇\2₉→ 6.3\5.2 ←2⁷\2⁹ 2₉\2₁₁→ 5.2\4.2 ←2⁹\2¹¹ 2₁₁\2₁→ 4.2\10 ←2¹¹\2¹

(3)

Musical staff showing horizontal crosses. The staff has two measures of music. The first measure consists of notes on the 3rd, 5th, and 7th steps of the scale. The second measure consists of notes on the 10th, 12th, and 14th steps. The notes are connected by horizontal lines between them.

3₁\3₃→ 2.1\7.3 ←3¹\3³ 3₃\3₅→ 7.3\6.3 ←3³\3⁵ 3₅\3₇→ 6.3\1.2 ←3⁵\3⁷

3₇\3₉→ 1.2\7.2 ←3⁷\3⁹ 3₉\3₁₁→ 7.2\5.1 ←3⁹\3¹¹ 3₁₁\3₁→ 5.1\2.1 ←3¹¹\3¹

(4)

Musical staff showing horizontal crosses. The staff has two measures of music. The first measure consists of notes on the 4th, 6th, and 8th steps of the scale. The second measure consists of notes on the 10th, 12th, and 14th steps. The notes are connected by horizontal lines between them.

4₁\4₃→ 4.1\6.2 ←4¹\4³ 4₃\4₅→ 6.2\1.1 ←4³\4⁵ 4₅\4₇→ 1.1\1.2 ←4⁵\4⁷

4₇\4₉→ 1.2\5.2 ←4⁷\4⁹ 4₉\4₁₁→ 5.2\1.3 ←4⁹\4¹¹ 4₁₁\4₁→ 1.3\4.1 ←4¹¹\4¹

(5)

Musical staff showing horizontal crosses. The staff has two measures of music. The first measure consists of notes on the 5th, 7th, and 9th steps of the scale. The second measure consists of notes on the 10th, 12th, and 14th steps. The notes are connected by horizontal lines between them.

5₆\5₃→ 7.2\3.1 ←5⁶\5³ 5₃\5₅→ 3.1\5.1 ←5³\5⁵ 5₅\5₇→ 5.1\7.3 ←5⁵\5⁷

5₇\5₉→ 7.3\9 ←5⁷\5⁹ 5₉\5₁₁→ 9\6.3 ←5⁹\5¹¹ 5₁₁\5₁6₁→ 6.3\7.2 ←5¹¹\5¹6¹

(6)

Musical staff showing horizontal crosses. The staff has two measures of music. The first measure consists of notes on the 5th, 7th, and 9th steps of the scale. The second measure consists of notes on the 10th, 12th, and 14th steps. The notes are connected by horizontal lines between them.

5₆₁\6₃→ 7.2\1.1 ←5⁶₁\6³ 6₃\6₅→ 1.1\5.1 ←6³\6⁵ 6₅\6₇→ 5.1\7.1 ←6⁵\6⁷

6₇\6₉→ 7.1\9 ←6⁷\6⁹ 6₉\5₁₁→ 9\6.1 ←6⁹\5¹¹ 6₁₁\5₁6₁→ 6.1\7.2 ←6¹¹\5¹6¹

* = cross point.

Horizontal Crosses 2

(1)

Musical staff for exercise 1. It consists of two staves of five-line music. The notes are represented by small circles with stems. Some notes have a small asterisk (*) to their left, indicating they are cross points. The staff includes various musical markings such as flats, sharps, and rests. Below the staff are ten pairs of numbers separated by a backslash, representing rhythmic values.

Below the staff:

- $1^3\backslash 1^1 \rightarrow$
- $6.2\backslash 10$
- $\leftarrow 1_3\backslash 1_1$
- $1^5 2^5 \backslash 1^3 \rightarrow$
- $1.3\backslash 6.2$
- $\leftarrow 1_5 2_5 \backslash 1_3$
- $1^7\backslash 1^5 2^5 \rightarrow$
- $2.2\backslash 1.3$
- $\leftarrow 1_7\backslash 1_5 2_5$
- $1^9\backslash 1^7 \rightarrow$
- $5.2\backslash 2.2$
- $\leftarrow 1_9\backslash 1_7$
- $1^{11}\backslash 1^9 \rightarrow$
- $1.1\backslash 5.2$
- $\leftarrow 1_{11}\backslash 1_9$
- $1^1\backslash 1^{11} \rightarrow$
- $10\backslash 1.1$
- $\leftarrow 1_1\backslash 1_{11}$

(2)

Musical staff for exercise 2. It consists of two staves of five-line music. The notes are represented by small circles with stems. Some notes have a small asterisk (*) to their left, indicating they are cross points. The staff includes various musical markings such as flats, sharps, and rests. Below the staff are ten pairs of numbers separated by a backslash, representing rhythmic values.

Below the staff:

- $2^3\backslash 2^1 \rightarrow$
- $8\backslash 10$
- $\leftarrow 2_3\backslash 2_1$
- $1^5 2^5 \backslash 2^3 \rightarrow$
- $1.3\backslash 8$
- $\leftarrow 1_5 2_5 \backslash 2_3$
- $2^7\backslash 1^5 2^5 \rightarrow$
- $6.3\backslash 1.3$
- $\leftarrow 2_7\backslash 1_5 2_5$
- $2^9\backslash 2^7 \rightarrow$
- $5.2\backslash 6.3$
- $\leftarrow 2_9\backslash 2_7$
- $2^{11}\backslash 2^9 \rightarrow$
- $4.2\backslash 5.2$
- $\leftarrow 2_{11}\backslash 2_9$
- $2^1\backslash 2^{11} \rightarrow$
- $10\backslash 4.2$
- $\leftarrow 2_1\backslash 2_{11}$

(3)

Musical staff for exercise 3. It consists of two staves of five-line music. The notes are represented by small circles with stems. Some notes have a small asterisk (*) to their left, indicating they are cross points. The staff includes various musical markings such as flats, sharps, and rests. Below the staff are ten pairs of numbers separated by a backslash, representing rhythmic values.

Below the staff:

- $3^3\backslash 3^1 \rightarrow$
- $7.3\backslash 2.1$
- $\leftarrow 3_3\backslash 3_1$
- $3^5\backslash 3^3 \rightarrow$
- $6.3\backslash 7.3$
- $\leftarrow 3_5\backslash 3_3$
- $3^7\backslash 3^5 \rightarrow$
- $1.2\backslash 6.3$
- $\leftarrow 3_7\backslash 3_5$
- $3^9\backslash 3^7 \rightarrow$
- $7.2\backslash 1.2$
- $\leftarrow 3_9\backslash 3_7$
- $3^{11}\backslash 3^9 \rightarrow$
- $5.1\backslash 7.2$
- $\leftarrow 3_{11}\backslash 3_9$
- $3^1\backslash 3^{11} \rightarrow$
- $2.1\backslash 5.1$
- $\leftarrow 3_1\backslash 3_{11}$

(4)

Musical staff for exercise 4. It consists of two staves of five-line music. The notes are represented by small circles with stems. Some notes have a small asterisk (*) to their left, indicating they are cross points. The staff includes various musical markings such as flats, sharps, and rests. Below the staff are ten pairs of numbers separated by a backslash, representing rhythmic values.

Below the staff:

- $4^3\backslash 4^1 \rightarrow$
- $6.2\backslash 4.1$
- $\leftarrow 4_3\backslash 4_1$
- $4^5\backslash 4^3 \rightarrow$
- $1.1\backslash 6.2$
- $\leftarrow 4_5\backslash 4_3$
- $4^7\backslash 4^5 \rightarrow$
- $1.2\backslash 1.1$
- $\leftarrow 4_7\backslash 4_5$
- $4^9\backslash 4^7 \rightarrow$
- $5.2\backslash 1.2$
- $\leftarrow 4_9\backslash 4_7$
- $4^{11}\backslash 4^9 \rightarrow$
- $1.3\backslash 5.2$
- $\leftarrow 4_{11}\backslash 4_9$
- $4^1\backslash 4^{11} \rightarrow$
- $4.1\backslash 1.3$
- $\leftarrow 4_1\backslash 4_{11}$

(5)

Musical staff for exercise 5. It consists of two staves of five-line music. The notes are represented by small circles with stems. Some notes have a small asterisk (*) to their left, indicating they are cross points. The staff includes various musical markings such as flats, sharps, and rests. Below the staff are ten pairs of numbers separated by a backslash, representing rhythmic values.

Below the staff:

- $5^3\backslash 5^1 6^1 \rightarrow$
- $3.1\backslash 7.2$
- $\leftarrow 5_3\backslash 5_1 6_1$
- $5^5\backslash 5^3 \rightarrow$
- $5.1\backslash 3.1$
- $\leftarrow 5_5\backslash 5_3$
- $5^7\backslash 5^5 \rightarrow$
- $7.3\backslash 5.1$
- $\leftarrow 5_7\backslash 5_5$
- $5^9\backslash 5^7 \rightarrow$
- $9\backslash 7.3$
- $\leftarrow 5_9\backslash 5_7$
- $5^{11}\backslash 5^9 \rightarrow$
- $6.3\backslash 9$
- $\leftarrow 5_{11}\backslash 5_9$
- $5^1 6^1 \backslash 5^{11} \rightarrow$
- $7.2\backslash 6.3$
- $\leftarrow 5_6\backslash 5_{11}$

(6)

Musical staff for exercise 6. It consists of two staves of five-line music. The notes are represented by small circles with stems. Some notes have a small asterisk (*) to their left, indicating they are cross points. The staff includes various musical markings such as flats, sharps, and rests. Below the staff are ten pairs of numbers separated by a backslash, representing rhythmic values.

Below the staff:

- $6^3\backslash 5^1 6^1 \rightarrow$
- $1.1\backslash 7.2$
- $\leftarrow 6_3\backslash 5_1 6_1$
- $6^5\backslash 6^3 \rightarrow$
- $5.1\backslash 1.1$
- $\leftarrow 6_5\backslash 6_3$
- $6^7\backslash 6^5 \rightarrow$
- $7.1\backslash 5.1$
- $\leftarrow 6_7\backslash 6_5$
- $6^9\backslash 6^7 \rightarrow$
- $9\backslash 7.1$
- $\leftarrow 6_9\backslash 6_7$
- $5^{11}\backslash 6^9 \rightarrow$
- $6.1\backslash 9$
- $\leftarrow 5_{11}\backslash 6_9$
- $5^1 6^1 \backslash 6^{11} \rightarrow$
- $7.2\backslash 6.1$
- $\leftarrow 5_6\backslash 6_{11}$

* = cross point.

Parallel Sides and Complements...

I

1 2 3 4 5 6

1₁ 1₁₂(4⁹) 1¹²(4₉) 1⁷ 2₁ 2₁₂ 2¹² 2⁷ 3₁ 3₁₂(5₅) 3¹²(5⁵) 3⁷
 10 11.1 2.2 10 5.1 6.3 2.1 3.2 1.2

4₁ (1⁹)4₁₂ (1₉)4¹² 4⁷ 5₁6₁ 3⁸5₁₂ 3₈5¹² 5⁷ 5₁6₁ 6₁₂ 6¹² 6⁷
 4.1 11.1 1.2 7.2 4.3 7.3 7.2 4.2 4.2 7.1

II

1 2 3 4 5 6

1₃ 1₂4₈ 1²4⁸ 1⁹ 2₃ 2₂ 2² 2⁹ 3₃ 3₂5⁶ 3²5₆ 3⁹
 6.2 11.3 5.2 8 6.1 5.2 7.3 4.3 7.2

4₃ 1₈4₂ 1⁸4² 4⁹ 5₃ (3⁹)5₂ (3⁹)5² 5⁹ 6₃ 6₂ 6² 6⁹
 6.2 11.3 5.2 3.1 3.2 9 1.1 1.3 1.3 9

III

1 2 3 4 5 6

1₅2₅ 1₄4₄ 1⁴4⁴ 1¹¹ 1₅2₅ 2₄ 2⁴ 2¹¹ 3₅ 3₄5¹⁰ 3⁴5₁₀ 3¹¹
 1.3 9 1.1 1.3 11.2 4.2 6.3 10 5.1

4₅ 1₄4₄ 1⁴4⁴ 4¹¹ 5₅ 3¹⁰5₄ 3₁₀5⁴ 5¹¹ 6₅ 6₄ 6⁴ 6¹¹
 1.1 9 1.3 5.1 10 6.3 5.1 11.2 6.1

*Complements' indeces are underlined.

...Parallel Sides and Complements

IV

1₇
2.2 1_{6(4³)} 1<sup>6(4₃)
11.1 1¹
10 2₇
6.3 2₆
5.1 2⁶
10 2¹
1.2 3₇
3_{6(5₁₁)}
3.2 3^{6(5¹¹)}
2.1</sup>

(4)

(5)

(6)

4₇
1.2 (1³)4₆
11.1 (1₃)4⁶
4¹
4.1 5₇
7.3 3²5₆
4.3 3₂5⁶
7.2 5¹6¹
7.2 6₇
7.1 6₆
4.2 6⁶
7.2 5¹6¹

V

(1)

(2)

(3)

1₉
5.2 1₈4₂
11.3 1⁸4²
1³
6.2 2₉
5.2 2₈
6.1 2⁸
8 2³
8 3₉
7.2 3₈5¹²
4.3 3⁸5₁₂
3³
7.3

(4)

(5)

(6)

4₉
5.2 1₂4₈
11.3 1²4⁸
4³
6.2 5₉
9 (3₃)5₈
3.2 (3³)5⁸
3.1 5³
3.1 6₉
9 6₈
1.3 6⁸
1.1 6³

VI

(1)

(2)

(3)

1₁₁
1.1 1₁₀4₁₀
9 1¹⁰4¹⁰
1⁵2⁵
1.3 2₁₁
4.2 2₁₀
11.2 2¹⁰
1.3 1⁵2⁵
1.3 3₁₁
5.1 3₁₀5⁴
10 3¹⁰5₄
3⁵
6.3

(4)

(5)

(6)

4₁₁
1.3 1₁₀4₁₀
9 1¹⁰4¹⁰
4⁵
1.1 5₁₁
6.3 3⁴5₁₀
10 3₄5¹⁰
5.1 5⁵
5.1 6₁₁
6.1 6₁₀
11.2 6¹⁰
5.1

5.2

Complements 1

(Horizontal Dodecads in 4)

1

1_{14_1} 1_{24_8} $1_{3(4^6)}$ 1_{44_4} $1_{54_{11}}$ $1_{6(4^3)}$ 1_{74_7} 1_{84_2} $1_{9(4^{12})}$ $1_{104_{10}}$ 1_{114_5} $1_{12(4^9)}$
 9 11.3 11.1 9 11.3 11.1 9 11.3 11.1 9 11.3 11.1

2

2_1 2_2 2_3 2_4 2_5 2_6 2_7 2_8 2_9 2_{10} 2_{11} 2_{12}
 11.2 6.1 5.1 11.2 6.1 5.1 11.2 6.1 5.1 11.2 6.1 5.1

3

3_{15^1} 3_{25^6} $3_{3(5_8)}$ $3_{45^{10}}$ 3_{55^3} $3_{6(5_{11})}$ 3_{75^7} $3_{85^{12}}$ $3_{9(5_2)}$ 3_{105^4} 3_{115^9} $3_{12(5_5)}$
 10 4.3 3.2 10 4.3 3.2 10 4.3 3.2 10 4.3 3.2

4

1_{14_1} 1_{84_2} $(1^6)4_3$ 1_{44_4} 1_{114_5} $(1^3)4_6$ 1_{74_7} 1_{24_8} $(1^{12})4_9$ $1_{104_{10}}$ $1_{54_{11}}$ $(1^9)4_{12}$
 9 11.3 11.1 9 11.3 11.1 9 11.3 11.1 9 11.3 11.1

5

3_{15_1} $(39)5_2$ 3^55_3 $3^{10}5_4$ $(3_{12})5_5$ 3^25_6 3^75_7 $(3_3)5_8$ $3^{11}5_9$ 3^45_{10} $(3_6)5_{11}$ 3^85_{12}
 10 3.2 4.3 10 3.2 4.3 10 3.2 4.3 10 3.2 4.3

6

6_1 6_2 6_3 6_4 6_5 6_6 6_7 6_8 6_9 6_{10} 6_{11} 6_{12}
 11.2 1.3 4.2 11.2 1.3 4.2 11.2 1.3 4.2 11.2 1.3 4.2

Complements 2

(Horizontal Dodecads in 4)

(1)

(2)

(3)

(4)

(5)

(6)

1⁴1 **1²4⁸** **1³(4₆)** **1⁴4⁴** **1⁵4¹¹** **1⁶(4₃)** **1⁷4⁷** **1⁸4²** **1⁹(4₁₂)** **1¹⁰4¹⁰** **1¹¹4⁵** **1¹²(4₉)**

9 11.3 11.1 9 11.3 11.1 9 11.3 11.1 9 11.3 11.1

2¹ **2²** **2³** **2⁴** **2⁵** **2⁶** **2⁷** **2⁸** **2⁹** **2¹⁰** **2¹¹** **2¹²**

11.2 6.1 5.1 11.2 6.1 5.1 11.2 6.1 5.1 11.2 6.1 5.1

3⁵₁ **3²5₆** **3³(5⁸)** **3⁴5₁₀** **3⁵5₃** **3⁶(5¹¹)** **3⁷5₇** **3⁸5₁₂** **3⁹(5²)** **3¹⁰5₄** **3¹¹5₉** **3¹²(5⁵)**

10 4.3 3.2 10 4.3 3.2 10 4.3 3.2 10 4.3 3.2

1⁴1 **1⁸4²** **(1₆)4³** **1⁴4⁴** **1¹¹4⁵** **(1₃)4⁶** **1⁷4⁷** **1²4⁸** **(1₁₂)4⁹** **1¹⁰4¹⁰** **1⁵4¹¹** **(1₉)4¹²**

9 11.3 11.1 9 11.3 11.1 9 11.3 11.1 9 11.3 11.1

3₅¹ **(3⁹)5²** **3₅5³** **3₁₀5⁴** **(3¹²)5⁵** **3₂5⁶** **3₇5⁷** **(3³)5⁸** **3₁₁5⁹** **3₄5¹⁰** **(3⁶)5¹¹** **3₈5¹²**

10 3.2 4.3 10 3.2 4.3 10 3.2 4.3 10 3.2 4.3

6¹ **6²** **6³** **6⁴** **6⁵** **6⁶** **6⁷** **6⁸** **6⁹** **6¹⁰** **6¹¹** **6¹²**

11.2 1.3 4.2 11.2 1.3 4.2 11.2 1.3 4.2 11.2 1.3 4.2

Odd Complements 1
(Horizontal Hexads in 4)

(\Leftrightarrow)

Odd Complements 2
(Horizontal Hexads in 4)

1

1₄₁ 1₃(4⁶) 1₅4₁₁ 1₇4₇ 1₉(4¹²) 1₁₁4₅
9 11.1 11.3 9 11.1 11.3

2

2₁ 2₃ 2₅ 2₇ 2₉ 2₁₁
11.2 5.1 6.1 11.2 5.1 6.1

3

3₅¹ 3₃(5⁸) 3₅5³ 3₇5⁷ 3₉(5₂) 3₁₁5⁹
10 3.2 4.3 10 3.2 4.3

4

1₄₁ (1⁶)4₃ 1₁₁4₅ 1₇4₇ (1¹²)4₉ 1₅4₁₁
9 11.1 11.3 9 11.1 11.3

5

3⁵₁ 3⁵5₃ (3₁₂)5₅ 3⁷5₇ 3¹¹5₉ (3₆)5₁₁
10 4.3 3.2 10 4.3 3.2

6

6₁ 6₃ 6₅ 6₇ 6₉ 6₁₁
11.2 4.2 1.3 11.2 4.2 1.3

1

1¹⁴¹ 1³(4₆) 1⁵4¹¹ 1⁷4⁷ 1⁹(4₁₂) 1¹¹4⁵
9 11.1 11.3 9 11.1 11.3

2

2¹ 2³ 2⁵ 2⁷ 2⁹ 2¹¹
11.2 5.1 6.1 11.2 5.1 6.1

3

3¹⁵₁ 3³(5⁸) 3⁵5₃ 3⁷5₇ 3⁹(5₂) 3¹¹5₉
10 3.2 4.3 10 3.2 4.3

4

1¹⁴¹ (1₆)4³ 1¹¹4⁵ 1⁷4⁷ (1₁₂)4⁹ 1⁵4¹¹
9 11.1 11.3 9 11.1 11.3

5

3₅¹ 3₅5³ (3¹²)5⁵ 3₇5⁷ 3₁₁5⁹ (3⁶)5¹¹
10 4.3 3.2 10 4.3 3.2

6

6¹ 6³ 6⁵ 6⁷ 6⁹ 6¹¹
11.2 4.2 1.3 11.2 4.2 1.3

Odd Complements 3

(\Leftrightarrow)

Odd Complements 4

①

1₄₁ 1₅4₁₁ 1₉(4¹²) 1₇4₇ 1₁₁4₅ 1₃(4⁶)

②

2₁ 2₅ 2₉ 2₇ 2₁₁ 2₃

11.2 6.1 5.1 11.2 6.1 5.1

③

3₁5¹ 3₅5³ 3₉(5₂) 3₇5⁷ 3₁₁5⁹ 3₃(5₈)

10 4.3 3.2 10 4.3 3.2

④

1₄₁ 1₁₁4₅ (1¹²)4₉ 1₇4₇ 1₅4₁₁ (1⁶)4₃

9 11.3 11.1 9 11.3 11.1

⑤

3₁5₁ (3₁₂)5₅ 3₇5₇ 3₁₁5₉ (3₆)5₁₁ 3₅5₃

10 3.2 4.3 10 3.2 4.3

⑥

6₁ 6₅ 6₉ 6₇ 6₁₁ 6₃

11.2 1.3 4.2 11.2 1.3 4.2

①

1¹⁴¹ 1⁵4¹¹ 1⁹(4₁₂) 1⁷4⁷ 1¹¹4⁵ 1³(4₆)

9 11.3 11.1 9 11.3 11.1

②

2¹ 2⁵ 2⁹ 2⁷ 2¹¹ 2³

11.2 6.1 5.1 11.2 6.1 5.1

③

3₁5₁ 3₅5₃ 3⁹(5₂) 3₇5₇ 3¹¹5₉ 3³(5₈)

10 4.3 3.2 10 4.3 3.2

④

1¹⁴¹ 1¹¹4⁵ (1₁₂)4⁹ 1⁷4⁷ 1⁵4¹¹ (1₆)4³

9 11.3 11.1 9 11.3 11.1

⑤

3₁5₁ (3₁₂)5₅ 3₇5₇ 3₁₁5₉ (3₆)5₁₁ 3₅5₃

10 3.2 4.3 10 3.2 4.3

⑥

6₁ 6₅ 6₉ 6₇ 6₁₁ 6₃

11.2 1.3 4.2 11.2 1.3 4.2

Even Complements 1
(Horizontal Hexads in 4)

Even Complements 2
(Horizontal Hexads in 4)

①

1248 1444 16(4³) 1842 1₁₀4₁₀ 1₁₂(4⁹)
11.3 9 11.1 11.3 9 11.1

②

2₂ 2₄ 2₆ 2₈ 2₁₀ 2₁₂
6.1 11.2 5.1 6.1 11.2 5.1

③

3₂5⁶ 3₄5¹⁰ 3₆(5₁₁) 3₈5¹² 3₁₀5⁴ 3₁₂(5₅)
4.3 10 3.2 4.3 10 3.2

④

1842 1444 (1³)4₆ 1248 1₁₀4₁₀ (1⁹)4₁₂
11.3 9 11.1 11.3 9 11.1

⑤

(3₉)5₂ 3₁₀5⁴ 3₂5⁶ (3₃)5₈ 3₄5₁₀ 3₈5₁₂
3.2 10 4.3 3.2 10 4.3

⑥

6₂ 6₄ 6₆ 6₈ 6₁₀ 6₁₂
1.3 11.2 4.2 1.3 11.2 4.2

①

1²4⁸ 1⁴4⁴ 1⁶(4₃) 1⁸4² 1¹⁰4¹⁰ 1¹²(4₉)
11.3 9 11.1 11.3 9 11.1

②

2² 2⁴ 2⁶ 2⁸ 2¹⁰ 2¹²
6.1 11.2 5.1 6.1 11.2 5.1

③

3²5₆ 3⁴5₁₀ 3⁶(5₁₁) 3⁸5₁₂ 3¹⁰5₄ 3¹²(5₅)
4.3 10 3.2 4.3 10 3.2

④

1⁸4² 1⁴4⁴ (1₃)4⁶ 1²4⁸ 1¹⁰4¹⁰ (1₉)4¹²
11.3 9 11.1 11.3 9 11.1

⑤

(3⁹)5² 3₁₀5⁴ 3₂5⁶ (3³)5⁸ 3₄5¹⁰ 3₈5¹²
3.2 10 4.3 3.2 10 4.3

⑥

6² 6⁴ 6⁶ 6⁸ 6¹⁰ 6¹²
1.3 11.2 4.2 1.3 11.2 4.2

Even Complements 3

(\Leftrightarrow)

Even Complements 4

①

$1_2 4_8$ $1_6(4^3)$ $1_{10} 4_{10}$ $1_8 4_2$ $1_{12}(4^9)$ $1_4 4_4$
11.3 11.1 9 11.3 11.1 9

②

2_2 2_6 2_{10} 2_8 2_{12} 2_4
6.1 5.1 11.2 6.1 5.1 11.2

③

3_{25}^6 $3_6(5_{11})$ $3_{10} 5^4$ $3_8 5^{12}$ $3_{12}(5_5)$ $3_4 5^{10}$
4.3 3.2 10 4.3 3.2 10

④

$1_8 4_2$ $(1^3) 4_6$ $1_{10} 4_{10}$ $1_2 4_8$ $(1^9) 4_{12}$ $1_4 4_4$
11.3 11.1 9 11.3 11.1 9

⑤

$(3_9) 5_2$ $3^2 5_6$ $3^4 5_{10}$ $(3_3) 5_8$ $3^8 5_{12}$ $3^{10} 5_4$
3.2 4.3 10 3.2 4.3 10

⑥

6_2 6_6 6_{10} 6_8 6_{12} 6_4
1.3 4.2 11.2 1.3 4.2 11.2

①

1^{24}^8 $1^6(4_3)$ $1^{10} 4^{10}$ $1^8 4^2$ $1^{12}(4_9)$ $1^4 4^4$
11.3 11.1 9 11.3 11.1 9

②

2^2 2^6 2^{10} 2^8 2^{12} 2^4
6.1 5.1 11.2 6.1 5.1 11.2

③

3^{25}^6 $3^6(5^{11})$ $3^{10} 5_4$ $3^8 5_{12}$ $3^{12}(5^5)$ $3^4 5_{10}$
4.3 3.2 10 4.3 3.2 10

④

$1^8 4^2$ $(1_3) 4^6$ $1^{10} 4^{10}$ $1^2 4^8$ $(1_9) 4^{12}$ $1^4 4^4$
11.3 11.1 9 11.3 11.1 9

⑤

$(3^9) 5^2$ $3_2 5^6$ $3_4 5^{10}$ $(3^3) 5^8$ $3_8 5^{12}$ $3_{10} 5^4$
3.2 4.3 10 3.2 4.3 10

⑥

6^2 6^6 6^{10} 6^8 6^{12} 6^4
1.3 4.2 11.2 1.3 4.2 11.2

Horizontal Parallel Sides and Complements...

(1) (2) (3) I (4) (5) (6)

1₁ → 10 ← 1¹ 2₁ → 10 ← 2¹ 3₁ → 2.1 ← 3¹ 4₁ → 4.1 ← 4¹ 5₁ 6₁ → 7.2 ← 5¹ 6¹ 5₁ 6₁ → 7.2 ← 5¹ 6¹

$\underline{1}_9(4^{12})$ 11.1 $\underline{1}^9(4_{12})$ 5.1 $\underline{2}_9$ 2⁹ $\underline{3}_9(5_2)$ 3.2 $\underline{3}^9(5^2)$ 11.1 $\underline{(1^{12})4}_9$ 4⁹ $\underline{3}^{11}5_9$ 4.3 $\underline{3}_{11}5^9$ 4.2 $\underline{6}_9$ 6⁹

1⁷ → 2.2 ← 1₇ 2⁷ → 6.3 ← 2₇ 3⁷ → 1.2 ← 3₇ 4⁷ → 1.2 ← 4₇ 5⁷ → 7.3 ← 5₇ 6⁷ → 7.1 ← 6₇

(1) (2) (3) II (4) (5) (6)

1₃ → 6.2 ← 1³ 2₃ → 8 ← 2³ 3₃ → 7.3 ← 3³ 4₃ → 6.2 ← 4³ 5₃ → 3.1 ← 5³ 6₃ → 1.1 ← 6³

$\underline{1}_{11}4_5$ 11.3 $\underline{1}^{11}4^5$ 6.1 $\underline{2}_{11}$ 2¹¹ $\underline{3}_{11}5^9$ 4.3 $\underline{3}^{11}5_9$ 11.3 $\underline{1}_54_{11}$ 11.3 $\underline{1}^54^{11}$ 3.2 $\underline{(3_6)5}_{11}$ 3.2 $\underline{(3^6)5}^{11}$ 1.3 $\underline{6}_{11}$ 6¹¹

1⁹ → 5.2 ← 1₉ 2⁹ → 5.2 ← 2₉ 3⁹ → 7.2 ← 3₉ 4⁹ → 5.2 ← 4₉ 5⁹ → 9 ← 5₉ 6⁹ → 9 ← 6₉

(1) (2) (3) III (4) (5) (6)

1₅ 2₅ → 1.3 ← 1⁵ 2⁵ 1₅ 2₅ → 1.3 ← 1⁵ 2⁵ 3₅ → 6.3 ← 3⁵ 4₅ → 1.1 ← 4⁵ 5₅ → 5.1 ← 5⁵ 6₅ → 5.1 ← 6⁵

$\underline{1}4_1$ 9 $\underline{1}^41$ 11.2 $\underline{2}_1$ 2¹ $\underline{3}_51$ 10 $\underline{3}^15_1$ 5.1 $\underline{1}_41$ 9 $\underline{1}^41$ 10 $\underline{3}_51$ 6₁ $\underline{3}^15_1$ 11.2

1¹¹ → 1.1 ← 1₁₁ 2¹¹ → 4.2 ← 2₁₁ 3¹¹ → 5.1 ← 3₁₁ 4¹¹ → 1.3 ← 4₁₁ 5¹¹ → 6.3 ← 5₁₁ 6¹¹ → 6.1 ← 6₁₁

*Complements' indeces are underlined.

...Horizontal Parallel Sides and Complements

(1) **(2)** **(3)** IV **(4)** **(5)** **(6)**

1₇→ 2.2 ←1⁷ 2₇→ 6.3 ←2⁷ 3₇→ 1.2 ←3⁷ 4₇→ 1.2 ←4⁷ 5₇→ 7.3 ←5⁷ 6₇→ 7.1 ←6⁷

$\frac{1_3(4^6)}{11.1}$ $\frac{1^3(4_6)}{5.1}$ $\frac{2^3}{3.2}$ $\frac{3_3(5_8)}{11.1}$ $\frac{3^3(5^8)}{(1^6)4_3}$ $\frac{(1_6)4^3}{4.3}$ $\frac{3^55_3}{3_55^3}$ $\frac{6_3}{4.2}$ $\frac{6^3}{}$

(1) **(2)** **(3)** V **(4)** **(5)** **(6)**

1¹→ 10 ←1₁ 2¹→ 10 ←2₁ 3¹→ 2.1 ←3₁ 4¹→ 4.1 ←4₁ 5¹6¹→ 7.2 ←5₁6₁ 5¹6¹→ 7.2 ←5₁6₁

$\frac{1_9}{5.2}$ $\frac{-1^9}{5.2}$ $\frac{2_9}{2^9}$ $\frac{3_9}{7.2}$ $\frac{-3^9}{}$ $\frac{4_9}{5.2}$ $\frac{-4^9}{}$ $\frac{5_9}{9}$ $\frac{-5^9}{}$ $\frac{6_9}{9}$ $\frac{-6^9}{}$

$\frac{1_54_{11}}{11.3}$ $\frac{1^{54}_{11}}{6.1}$ $\frac{2_5}{2^5}$ $\frac{3_55^3}{4.3}$ $\frac{3^55_3}{11.3}$ $\frac{1_{11}4_5}{11.3}$ $\frac{1^{11}4^5}{}$ $\frac{(3_{12})5_5}{3.2}$ $\frac{(3^{12})5^5}{1.3}$ $\frac{6_5}{6^5}$

(1) **(2)** **(3)** VI **(4)** **(5)** **(6)**

1₁₁→ 1.1 ←1¹¹ 2₁₁→ 4.2 ←2¹¹ 3₁₁→ 5.1 ←3¹¹ 4₁₁→ 1.3 ←4¹¹ 5₁₁→ 6.3 ←5¹¹ 6₁₁→ 6.1 ←6¹¹

$\frac{1_{14}}{9}$ $\frac{1^{14}}{11.2}$ $\frac{2_7}{2^7}$ $\frac{3_57}{10}$ $\frac{3^75_7}{}$ $\frac{1_{14}}{9}$ $\frac{1^{14}}{10}$ $\frac{3^75_7}{10}$ $\frac{3_57}{11.2}$

$\frac{1^{52^5}}{1.3}$ $\frac{-1_{52^5}}{1.3}$ $\frac{1^{52^5}}{6.3}$ $\frac{-1_{52^5}}{6.3}$ $\frac{4^5}{1.1}$ $\frac{-4_5}{}$ $\frac{5^5}{5.1}$ $\frac{-5_5}{5.1}$ $\frac{6^5}{5.1}$ $\frac{-6_5}{5.1}$

Horizontal Complements 1

(1)

Musical staff 1 consists of two staves. The top staff has a treble clef and the bottom staff has a bass clef. Both staves have a key signature of one flat. The music is in common time. The notes are represented by dots and stems. Below each staff is a series of numbered labels indicating specific notes or intervals.

Top Staff Notes:

- $1_{14_1} \rightarrow 9$
- $\leftarrow 1_{74_7}$
- $1_{24_8} \rightarrow 11.3$
- $\leftarrow 1_{84_2}$
- $1_3(4^6) \rightarrow 11.1$
- $\leftarrow 1_9(4^{12})$
- $1_{44_4} \rightarrow 9$
- $\leftarrow 1_{104_{10}}$
- $1_{54_{11}} \rightarrow 11.3$
- $\leftarrow 1_{114_5}$
- $1_6(4^3) \rightarrow 11.1$
- $\leftarrow 1_{12}(4^9)$

Bottom Staff Notes:

- $1_{74_7} \rightarrow 9$
- $\leftarrow 1_{14_1}$
- $1_{84_2} \rightarrow 11.3$
- $\leftarrow 1_{24_8}$
- $1_9(4^{12}) \rightarrow 11.1$
- $\leftarrow 1_{13}(4^6)$
- $1_{104_{10}} \rightarrow 9$
- $\leftarrow 1_{44_4}$
- $1_{114_5} \rightarrow 11.3$
- $\leftarrow 1_{54_{11}}$
- $1_{12}(4^9) \rightarrow 11.1$
- $\leftarrow 1_{6}(4^3)$

(2)

Musical staff 2 consists of two staves. The top staff has a treble clef and the bottom staff has a bass clef. Both staves have a key signature of one flat. The music is in common time. The notes are represented by dots and stems. Below each staff is a series of numbered labels indicating specific notes or intervals.

Top Staff Notes:

- $2_1 \rightarrow 11.2$
- $\leftarrow 2_7$
- $2_2 \rightarrow 6.1$
- $\leftarrow 2_8$
- $2_3 \rightarrow 5.1$
- $\leftarrow 2_9$
- $2_4 \rightarrow 11.2$
- $\leftarrow 2_{10}$
- $2_5 \rightarrow 6.1$
- $\leftarrow 2_{11}$
- $2_6 \rightarrow 5.1$
- $\leftarrow 2_{12}$

Bottom Staff Notes:

- $2_7 \rightarrow 11.2$
- $\leftarrow 2_1$
- $2_8 \rightarrow 6.1$
- $\leftarrow 2_2$
- $2_9 \rightarrow 5.1$
- $\leftarrow 2_3$
- $2_{10} \rightarrow 11.2$
- $\leftarrow 2_4$
- $2_{11} \rightarrow 6.1$
- $\leftarrow 2_5$
- $2_{12} \rightarrow 5.1$
- $\leftarrow 2_6$

(3)

Musical staff 3 consists of two staves. The top staff has a treble clef and the bottom staff has a bass clef. Both staves have a key signature of one flat. The music is in common time. The notes are represented by dots and stems. Below each staff is a series of numbered labels indicating specific notes or intervals.

Top Staff Notes:

- $3_{15^1} \rightarrow 10$
- $\leftarrow 3_{75^7}$
- $3_{25^6} \rightarrow 4.3$
- $\leftarrow 3_{85^{12}}$
- $3_3(5_8) \rightarrow 3.2$
- $\leftarrow 3_9(5_2)$
- $3_{45^{10}} \rightarrow 10$
- $\leftarrow 3_{105^4}$
- $3_{55^3} \rightarrow 4.3$
- $\leftarrow 3_{115^9}$
- $3_6(5_{11}) \rightarrow 3.2$
- $\leftarrow 3_{12}(5_5)$

Bottom Staff Notes:

- $3_{75^7} \rightarrow 10$
- $\leftarrow 3_{15^1}$
- $3_{85^{12}} \rightarrow 4.3$
- $\leftarrow 3_{25^6}$
- $3_0(5_2) \rightarrow 3.2$
- $\leftarrow 3_3(5_8)$
- $3_{105^4} \rightarrow 10$
- $\leftarrow 3_{45^{10}}$
- $3_{115^9} \rightarrow 4.3$
- $\leftarrow 3_{55^3}$
- $3_{12}(5_5) \rightarrow 3.2$
- $\leftarrow 3_6(5_{11})$

(4)

Musical staff 4 consists of two staves. The top staff has a treble clef and the bottom staff has a bass clef. Both staves have a key signature of one flat. The music is in common time. The notes are represented by dots and stems. Below each staff is a series of numbered labels indicating specific notes or intervals.

Top Staff Notes:

- $1_{14_1} \rightarrow 9$
- $\leftarrow 1_{74_7}$
- $1_{84_2} \rightarrow 11.3$
- $\leftarrow 1_{24_8}$
- $(1^6)4_3 \rightarrow 11.1$
- $\leftarrow (1^{12})4_9$
- $1_{44_4} \rightarrow 9$
- $\leftarrow 1_{104_{10}}$
- $1_{114_5} \rightarrow 11.3$
- $\leftarrow 1_{54_{11}}$
- $(1^3)4_6 \rightarrow 11.1$
- $\leftarrow (1^9)4_{12}$

Bottom Staff Notes:

- $1_{74_7} \rightarrow 9$
- $\leftarrow 1_{14_1}$
- $1_{24_8} \rightarrow 11.3$
- $\leftarrow 1_{84_2}$
- $(1^{12})4_9 \rightarrow 11.1$
- $\leftarrow (1^6)4_3$
- $1_{104_{10}} \rightarrow 9$
- $\leftarrow 1_{44_4}$
- $1_{54_{11}} \rightarrow 11.3$
- $\leftarrow 1_{114_5}$
- $(1^9)4_{12} \rightarrow 11.1$
- $\leftarrow (1^3)4_6$

(5)

Musical staff 5 consists of two staves. The top staff has a treble clef and the bottom staff has a bass clef. Both staves have a key signature of one flat. The music is in common time. The notes are represented by dots and stems. Below each staff is a series of numbered labels indicating specific notes or intervals.

Top Staff Notes:

- $3^{15_1} \rightarrow 10$
- $\leftarrow 3^{75_7}$
- $(3_9)5_2 \rightarrow 3.2$
- $\leftarrow (3_3)5_8$
- $3^{55_3} \rightarrow 4.3$
- $\leftarrow 3^{115_9}$
- $3^{105_4} \rightarrow 10$
- $\leftarrow 3^{45_{10}}$
- $(3_{12})5_5 \rightarrow 3.2$
- $\leftarrow (3_6)5_{11}$
- $3^{25_6} \rightarrow 4.3$
- $\leftarrow 3^{85_{12}}$

Bottom Staff Notes:

- $3^{75_7} \rightarrow 10$
- $\leftarrow 3^{15_1}$
- $(3_3)5_8 \rightarrow 3.2$
- $\leftarrow (3_9)5_2$
- $3^{115_9} \rightarrow 4.3$
- $\leftarrow 3^{55_3}$
- $3^{45_{10}} \rightarrow 10$
- $\leftarrow 3^{105_4}$
- $(3_6)5_{11} \rightarrow 3.2$
- $\leftarrow (3_{12})5_5$
- $3^{85_{12}} \rightarrow 4.3$
- $\leftarrow 3^{25_6}$

(6)

Musical staff 6 consists of two staves. The top staff has a treble clef and the bottom staff has a bass clef. Both staves have a key signature of one flat. The music is in common time. The notes are represented by dots and stems. Below each staff is a series of numbered labels indicating specific notes or intervals.

Top Staff Notes:

- $6_1 \rightarrow 11.2$
- $\leftarrow 6_7$
- $6_2 \rightarrow 1.3$
- $\leftarrow 6_8$
- $6_3 \rightarrow 4.2$
- $\leftarrow 6_9$
- $6_4 \rightarrow 11.2$
- $\leftarrow 6_{10}$
- $6_5 \rightarrow 1.3$
- $\leftarrow 6_{11}$
- $6_6 \rightarrow 4.2$
- $\leftarrow 6_{12}$

Bottom Staff Notes:

- $6_7 \rightarrow 11.2$
- $\leftarrow 6_1$
- $6_8 \rightarrow 1.3$
- $\leftarrow 6_2$
- $6_9 \rightarrow 4.2$
- $\leftarrow 6_3$
- $6_{10} \rightarrow 11.2$
- $\leftarrow 6_4$
- $6_{11} \rightarrow 1.3$
- $\leftarrow 6_5$
- $6_{12} \rightarrow 4.2$
- $\leftarrow 6_6$

5.11

Horizontal Complements 2

(1)

Musical staff 1 consists of two staves. The top staff has notes with stems pointing right, and the bottom staff has notes with stems pointing left. Various note heads are marked with black dots, open circles, or filled circles. Below each staff are numerical labels indicating specific note positions or complements.

Top staff notes: 1⁴₁ → 9, ← 1⁷₄⁷, 1²₄⁸ → 11.3, ← 1⁸₄², 1³(4₆) → 11.1, ← 1⁹(4₁₂) → 11.3, 1⁴₄⁴ → 9, ← 1¹⁰₄¹⁰, 1⁵₄¹¹ → 11.3, ← 1¹¹₄⁵, 1⁶(4₃) → 11.1, ← 1¹²(4₉) → 11.1.

Bottom staff notes: 1⁷₄⁷ → 9, ← 1¹₄¹, 1⁸₄² → 11.3, ← 1²₄⁸, 1⁹(4₁₂) → 11.1, ← 1³(4₆) → 11.1, 1¹⁰₄¹⁰ → 9, ← 1⁴₄⁴, 1¹¹₄⁵ → 11.3, ← 1⁵₄¹¹, 1¹²(4₉) → 11.1, ← 1⁶(4₃) → 11.1.

(2)

Musical staff 2 consists of two staves. The top staff has notes with stems pointing right, and the bottom staff has notes with stems pointing left. Numerical labels below the staves indicate specific note positions.

Top staff notes: 2¹ → 11.2, ← 2⁷, 2² → 6.1, ← 2⁸, 2³ → 5.1, ← 2⁹, 2⁴ → 11.2, ← 2¹⁰, 2⁵ → 6.1, ← 2¹¹, 2⁶ → 5.1, ← 2¹².

Bottom staff notes: 2⁷ → 11.2, ← 2¹, 2⁸ → 6.1, ← 2², 2⁹ → 5.1, ← 2³, 2¹⁰ → 11.2, ← 2⁴, 2¹¹ → 6.1, ← 2⁵, 2¹² → 5.1, ← 2⁶.

(3)

Musical staff 3 consists of two staves. The top staff has notes with stems pointing right, and the bottom staff has notes with stems pointing left. Numerical labels below the staves indicate specific note positions.

Top staff notes: 3¹₅₁ → 10, ← 3⁷₅₇, 3²₅₆ → 4.3, ← 3⁸₅₁₂, 3³(5⁸) → 3.2, ← 3⁹(5²), 3⁴₅₁₀ → 10, ← 3¹⁰₅₄, 3⁵₅₃ → 4.3, ← 3¹¹₅₉, 3⁶(5¹¹) → 3.2, ← 3¹²(5⁵).

Bottom staff notes: 3⁷₅₇ → 10, ← 3¹₅₁, 3⁸₅₁₂ → 4.3, ← 3²₅₆, 3⁹(5²) → 3.2, ← 3³(5⁸), 3¹⁰₅₄ → 10, ← 3⁴₅₁₀, 3¹¹₅₉ → 4.3, ← 3⁵₅₃, 3¹²(5⁵) → 3.2, ← 3⁶(5¹¹).

(4)

Musical staff 4 consists of two staves. The top staff has notes with stems pointing right, and the bottom staff has notes with stems pointing left. Numerical labels below the staves indicate specific note positions.

Top staff notes: 1⁴₁ → 9, ← 1⁷₄⁷, 1⁸₄² → 11.3, ← 1²₄⁸, (1₆)⁴³ → 11.1, ← (1₁₂)⁴⁹, 1⁴₄⁴ → 9, ← 1¹⁰₄¹⁰, 1¹¹₄⁵ → 11.3, ← 1⁵₄¹¹, (1₃)⁴⁶ → 11.1, ← (1₉)⁴¹².

Bottom staff notes: 1⁷₄⁷ → 9, ← 1¹₄¹, 1²₄⁸ → 11.3, ← 1⁸₄², (1₁₂)⁴⁹ → 11.1, ← (1₆)⁴³, 1¹⁰₄¹⁰ → 9, ← 1⁴₄⁴, 1⁵₄¹¹ → 11.3, ← 1¹¹₄⁵, (1₉)⁴¹² → 11.1, ← (1₃)⁴⁶.

(5)

Musical staff 5 consists of two staves. The top staff has notes with stems pointing right, and the bottom staff has notes with stems pointing left. Numerical labels below the staves indicate specific note positions.

Top staff notes: 3₁⁵ → 10, ← 3₇⁵₇, (3⁹)⁵² → 3.2, ← (3³)⁵⁸, 3₅⁵₃ → 4.3, ← 3₁₁⁵₉, 3₁₀⁵₄ → 10, ← 3₄⁵₁₀, (3¹²)⁵⁵ → 3.2, ← (3⁶)⁵¹¹, 3₂⁵₆ → 4.3, ← 3₈⁵₁₂.

Bottom staff notes: 3₅⁷ → 10, ← 3₁⁵₁, (3³)⁵⁸ → 3.2, ← (3⁹)⁵², 3₁₁⁵₉ → 4.3, ← 3₅⁵₃, 3₄⁵₁₀ → 10, ← 3₁₀⁵₄, (3⁶)⁵¹¹ → 3.2, ← (3¹²)⁵⁵, 3₈⁵₁₂ → 4.3, ← 3₂⁵₆.

(6)

Musical staff 6 consists of two staves. The top staff has notes with stems pointing right, and the bottom staff has notes with stems pointing left. Numerical labels below the staves indicate specific note positions.

Top staff notes: 6¹ → 11.2, ← 6⁷, 6² → 1.3, ← 6⁸, 6³ → 4.2, ← 6⁹, 6⁴ → 11.2, ← 6¹⁰, 6⁵ → 1.3, ← 6¹¹, 6⁶ → 4.2, ← 6¹².

Bottom staff notes: 6⁷ → 11.2, ← 6¹, 6⁸ → 1.3, ← 6², 6⁹ → 4.2, ← 6³, 6¹⁰ → 11.2, ← 6⁴, 6¹¹ → 1.3, ← 6⁵, 6¹² → 4.2, ← 6⁶.

5.12

Outer Triads 1

(Horizontal Outer Hexads in 3)

(1)

(2)

(3)

(4)

(5)

(6)

Outer Triads 2

(Horizontal Outer Hexads in 3)

(1)

(2)

(3)

(4)

(5)

(6)

6.2

Inner Triads 1

(1)

1₄2₂ 1₁₀2⁴ 1₆2₆ 1₁₂2¹² 1₈2₈ 1₂2¹⁰ 1₄2₂ 1₁₀2⁴ 1₆2₆ 1₁₂2¹² 1₈2₈ 1₂2¹⁰
2 6.1 2 6.1 2 6.1 2 6.1 2 6.1 2 6.1 2 6.1

(2)

1₄2₂ 1²2₁₀ 1₆2₆ 1¹²2₁₂ 1₈2₈ 1¹⁰2₄ 1₄2₂ 1²2₁₀ 1₆2₆ 1¹²2₁₂ 1₈2₈ 1¹⁰2₄
2 6.1 2 6.1 2 6.1 2 6.1 2 6.1 2 6.1 2 6.1

(3)

3₄ 3₁₀ 3₆ 3₁₂ 3₈ 3₂ 3₄ 3₁₀ 3₆ 3₁₂ 3₈ 3₂
10.1 12 10.1 12 10.1 12 10.1 12 10.1 12 10.1 12

(4)

4₄ 4₁₀ 4₆ 4₁₂ 4₈ 4₂ 4₄ 4₁₀ 4₆ 4₁₂ 4₈ 4₂
6.2 4 6.2 4 6.2 4 6.2 4 6.2 4 6.2 4

(5)

5₁₀6⁶ 5₁₂6₁₀ 5₆6¹² 5₂6₂ 5₈6⁸ 5₄6₄ 5₁₀6⁶ 5₁₂6₁₀ 5₆6¹² 5₂6₂ 5₈6⁸ 5₄6₄
10.2 8 10.2 8 10.2 8 10.2 8 10.2 8 10.2 8

(6)

5⁶6₁₂ 5₁₂6₁₀ 5¹⁰6₆ 5₂6₂ 5⁸6₈ 5₄6₄ 5⁶6₁₂ 5₁₂6₁₀ 5¹⁰6₆ 5₂6₂ 5⁸6₈ 5₄6₄
10.2 8 10.2 8 10.2 8 10.2 8 10.2 8 10.2 8
6.3

Inner Triads 2

(1)

C||

1⁴2² 1¹⁰2₄ 1⁶2⁶ 1¹²2₁₂ 182⁸ 1²2₁₀ 1⁴2² 1¹⁰2₄ 1⁶2⁶ 1¹²2₁₂ 182⁸ 1²2₁₀

2 6.1 2 6.1 2 6.1 2 6.1 2 6.1 2 6.1

(2)

C||

1⁴2² 1₂2¹⁰ 1⁶2⁶ 1₁₂2¹² 182⁸ 1₁₀2⁴ 1⁴2² 1₂2¹⁰ 1⁶2⁶ 1₁₂2¹² 182⁸ 1₁₀2⁴

2 6.1 2 6.1 2 6.1 2 6.1 2 6.1 2 6.1

(3)

C||

3⁴ 3¹⁰ 3⁶ 3¹² 3⁸ 3² 3⁴ 3¹⁰ 3⁶ 3¹² 3⁸ 3²

10.1 12 10.1 12 10.1 12 10.1 12 10.1 12 10.1 12

(4)

C||

4⁴ 4¹⁰ 4⁶ 4¹² 4⁸ 4² 4⁴ 4¹⁰ 4⁶ 4¹² 4⁸ 4²

6.2 4 6.2 4 6.2 4 6.2 4 6.2 4 6.2 4

(5)

C||

5¹⁰6₆ 5¹²6¹⁰ 5⁶6₁₂ 5²6² 5⁸6₈ 5⁴6⁴ 5¹⁰6₆ 5¹²6¹⁰ 5⁶6₁₂ 5²6² 5⁸6₈ 5⁴6⁴

10.2 8 10.2 8 10.2 8 10.2 8 10.2 8 10.2 8

(6)

C||

5₆6¹² 5¹²6¹⁰ 5₁₀6⁶ 5²6² 5₈6⁸ 5⁴6⁴ 5₆6¹² 5¹²6¹⁰ 5₁₀6⁶ 5²6² 5₈6⁸ 5⁴6⁴

10.2 8 10.2 8 10.2 8 10.2 8 10.2 8 10.2 8 6.4

Outer and Inner Triads 1

Outer and Inner Triads 2

(1)

Musical staff showing outer and inner triad patterns for exercise 1. The staff has two systems of six measures each. Measures 1-3 show outer triads (triads on the outer voices) and measures 4-6 show inner triads (triads on the inner voices). The key signature is C major.

1¹2¹ 1⁴2² 1³2₃ 1¹⁰2₄ 1⁵2⁵ 1⁶2⁶ 1⁷2₁₁ 1¹²2₁₂ 1⁹2⁹ 1⁸2⁸ 1¹¹2₇ 1²2₁₀
5.2 2 3 6.1 5.2 2 3 6.1 5.2 2 3 6.1

(2)

Musical staff showing outer and inner triad patterns for exercise 2. The staff has two systems of six measures each. Measures 1-3 show outer triads and measures 4-6 show inner triads. The key signature is C major.

1¹2¹ 1⁴2² 1₃2³ 1₂2¹⁰ 1⁵2⁵ 1⁶2⁶ 1₁₁2⁷ 1₁₂2¹² 1⁹2⁹ 1⁸2⁸ 1₇2¹¹ 1₁₀2⁴
5.2 2 3 6.1 5.2 2 3 6.1 5.2 2 3 6.1

(3)

Musical staff showing outer and inner triad patterns for exercise 3. The staff has two systems of six measures each. Measures 1-3 show outer triads and measures 4-6 show inner triads. The key signature is C major.

3¹ 3⁴ 3³ 3¹⁰ 3⁵ 3⁶ 3⁷ 3¹² 3⁹ 3⁸ 3¹¹ 3² 3¹ 3⁴ 3³ 3¹⁰ 3⁵ 3⁶ 3⁷ 3¹² 3⁹ 3⁸ 3¹¹ 3²
9.2 10.1 7 12 9.2 10.1 7 12 9.2 10.1 7 12 9.2 10.1 7 12 9.2 10.1 7 12 9.2 10.1 7 12

(4)

Musical staff showing outer and inner triad patterns for exercise 4. The staff has two systems of six measures each. Measures 1-3 show outer triads and measures 4-6 show inner triads. The key signature is C major.

4¹ 4⁴ 4³ 4¹⁰ 4⁵ 4⁶ 4⁷ 4¹² 4⁹ 4⁸ 4¹¹ 4² 4¹ 4⁴ 4³ 4¹⁰ 4⁵ 4⁶ 4⁷ 4¹² 4⁹ 4⁸ 4¹¹ 4²
1 6.2 5.1 4 1 6.2 5.1 4 1 6.2 5.1 4 1 6.2 5.1 4 1 6.2 5.1 4 1 6.2 5.1 4

(5)

Musical staff showing outer and inner triad patterns for exercise 5. The staff has two systems of six measures each. Measures 1-3 show outer triads and measures 4-6 show inner triads. The key signature is C major.

5¹6¹ 5¹⁰6₆ 5³6₇ 5¹²6¹⁰ 5⁵6⁵ 5⁶6₁₂ 5⁷6₃ 5²6² 5⁹6⁹ 5⁸6₈ 5¹¹6₁₁ 5⁴6⁴
9.1 10.2 11 8 9.1 10.2 11 8 9.1 10.2 11 8 9.1 10.2 11 8 9.1 10.2 11 8 9.1 10.2 11 8

(6)

Musical staff showing outer and inner triad patterns for exercise 6. The staff has two systems of six measures each. Measures 1-3 show outer triads and measures 4-6 show inner triads. The key signature is C major.

5¹6¹ 5₆6¹² 5₇6³ 5¹²6¹⁰ 5⁵6⁵ 5₁₀6⁶ 5₃6⁷ 5²6² 5⁹6⁹ 5₈6⁸ 5₁₁6¹¹ 5⁴6⁴
9.1 10.2 11 8 9.1 10.2 11 8 9.1 10.2 11 8 9.1 10.2 11 8 9.1 10.2 11 8

Outer and Inner Equilateral Triangles 1

(Horizontal Dodecads in 3)

1

1₁2₁ 1₂ 1₃2³ 1₄ 1₅2₅ 1₆ 1₇2¹¹ 1₈ 1₉2₉ 1₁₀ 1₁₁2⁷ 1₁₂ 1₁2₁ 1₂ 1₃2³ 1₄ 1₅2₅ 1₆ 1₇2¹¹ 1₈ 1₉2₉ 1₁₀ 1₁₁2⁷ 1₁₂

5.2 3 5.2 3 5.2 3 5.2 3 5.2 3 5.2 3 5.2 3 5.2 3 5.2 3 5.2 3 5.2 3

2

1₁2₁ 2₂ 1³2₃ 2₄ 1₅2₅ 2₆ 1¹¹2₇ 2₈ 1₉2₉ 2₁₀ 1⁷2₁₁ 2₁₂ 1₁2₁ 2₂ 1³2₃ 2₄ 1₅2₅ 2₆ 1¹¹2₇ 2₈ 1₉2₉ 2₁₀ 1⁷2₁₁ 2₁₂

5.2 3 5.2 3 5.2 3 5.2 3 5.2 3 5.2 3 5.2 3 5.2 3 5.2 3 5.2 3 5.2 3

3

3₁ 3₂ 3₃ 3₄ 3₅ 3₆ 3₇ 3₈ 3₉ 3₁₀ 3₁₁ 3₁₂ 3₁ 3₂ 3₃ 3₄ 3₅ 3₆ 3₇ 3₈ 3₉ 3₁₀ 3₁₁ 3₁₂

9.2 7 9.2 7 9.2 7 9.2 7 9.2 7 9.2 7 9.2 7 9.2 7 9.2 7 9.2 7 9.2 7

4

4₁ 4₂ 4₃ 4₄6₁₂ 4₅ 4₆ 4₇ 4₈6₄ 4₉ 4₁₀ 4₁₁ 4₁₂6₈ 4₁ 4₂ 4₃ 4₄6₁₂ 4₅ 4₆ 4₇ 4₈6₄ 4₉ 4₁₀ 4₁₁ 4₁₂6₈

1 5.1 1 5.1 1 5.1 1 5.1 1 5.1 1 5.1 1 5.1 1 5.1 1 5.1

5

5₁6₁ 5₂ 5₃6⁷ 5₄ 5₅6₅ 5₆ 5₇6³ 5₈ 5₉6₉ 5₁₀ 5₁₁6¹¹ 5₁₂ 5₁6₁ 5₂ 5₃6⁷ 5₄ 5₅6₅ 5₆ 5₇6³ 5₈ 5₉6₉ 5₁₀ 5₁₁6¹¹ 5₁₂

9.1 11 9.1 11 9.1 11 9.1 11 9.1 11 9.1 11 9.1 11 9.1 11 9.1 11

6

5₁6₁ 6₂ 5⁷6₃ 4₈6₄ 5₅6₅ 6₆ 5³6₇ 4₁₂6₈ 5₉6₉ 6₁₀ 5¹¹6₁₁ 4₄6₁₂ 5₁6₁ 6₂ 5⁷6₃ 4₈6₄ 5₅6₅ 6₆ 5³6₇ 4₁₂6₈ 5₉6₉ 6₁₀ 5¹¹6₁₁ 4₄6₁₂

9.1 11 9.1 11 9.1 11 9.1 11 9.1 11 9.1 11 9.1 11 9.1 11 9.1 11

Outer and Inner Equilateral Triangles 2

(Horizontal Dodecad in 3)

1 (↔)

1¹2¹ 1² 1³2₃ 1⁴ 1⁵2⁵ 1⁶ 1⁷2₁₁ 1⁸ 1⁹2⁹ 1¹⁰ 1¹¹2₇ 1¹²

5.2 3 5.2 3 5.2 3 5.2 3 5.2 3 5.2 3 5.2 3

2 (↔)

1¹2¹ 2² 1₃2³ 2⁴ 1⁵2⁵ 2⁶ 1₁₁2⁷ 2⁸ 1⁹2⁹ 2¹⁰ 1₇2¹¹ 2¹²

5.2 3 5.2 3 5.2 3 5.2 3 5.2 3 5.2 3 5.2 3

3 (↔)

3¹ 3² 3³ 3⁴ 3⁵ 3⁶ 3⁷ 3⁸ 3⁹ 3¹⁰ 3¹¹ 3¹²

9.2 7 9.2 7 9.2 7 9.2 7 9.2 7 9.2 7 9.2

4 (↔)

4¹ 4² 4³ 4⁴6¹² 4⁵ 4⁶ 4⁷ 4⁸6⁴ 4⁹ 4¹⁰ 4¹¹ 4¹²6⁸

1 5.1 1 5.1 1 5.1 1 5.1 1 5.1 1 5.1 1 5.1

5 (↔)

5¹6¹ 5² 5³6₇ 5⁴ 5⁵6⁵ 5⁶ 5⁷6₃ 5⁸ 5⁹6⁹ 5¹⁰ 5¹¹6₁₁ 5¹²

9.1 11 9.1 11 9.1 11 9.1 11 9.1 11 9.1 11

6 (↔)

5¹6¹ 6² 5⁷6³ 4⁸6⁴ 5⁵6⁵ 6⁶ 5₃6⁷ 4¹²6⁸ 5⁹6⁹ 6¹⁰ 5₁₁6¹¹ 4⁴6¹²

9.1 11 9.1 11 9.1 11 9.1 11 9.1 11 9.1 11

Horizontal Triads 1

(1)

Musical staff 1 consists of two staves. The top staff has a treble clef and a key signature of one flat. The bottom staff has a treble clef and a key signature of one sharp. The notes are represented by dots on the staff. Below each staff is a series of numbered intervals and their inversions, indicating the horizontal movement between notes.

Top Staff Intervals:
 1₄2₂ → 2 ← 1⁴2² 1₁₀2₄ → 6.1 ← 1¹⁰2₄ 1₆2₆ → 2 ← 1⁶2⁶ 1₁₂2₁₂ → 6.1 ← 1¹²2₁₂ 1₈2₈ → 2 ← 1⁸2⁸ 1₂2₁₀ → 6.1 ← 1²2₁₀

Bottom Staff Intervals:
 1₁2₁ → 5.2 ← 1¹2¹ 1₃2₃ → 3 ← 1³2₃ 1₅2₅ → 5.2 ← 1⁵2⁵ 1₇2¹¹ → 3 ← 1⁷2₁₁ 1₉2₉ → 5.2 ← 1⁹2⁹ 1₁₁2₇ → 3 ← 1¹¹2₇

(2)

Musical staff 2 consists of two staves. The top staff has a treble clef and a key signature of one flat. The bottom staff has a treble clef and a key signature of one sharp. The notes are represented by dots on the staff. Below each staff is a series of numbered intervals and their inversions, indicating the horizontal movement between notes.

Top Staff Intervals:
 1₄2₂ → 2 ← 1⁴2² 1²2₁₀ → 6.1 ← 1₂2¹⁰ 1₆2₆ → 2 ← 1⁶2⁶ 1¹²2₁₂ → 6.1 ← 1₁₂2₁₂ 1₈2₈ → 2 ← 1⁸2⁸ 1¹⁰2₄ → 6.1 ← 1₁₀2⁴

Bottom Staff Intervals:
 1₁2₁ → 5.2 ← 1¹2¹ 1³2₃ → 3 ← 1₃2³ 1₅2₅ → 5.2 ← 1⁵2⁵ 1¹¹2₇ → 3 ← 1₁₁2⁷ 1₉2₉ → 5.2 ← 1⁹2⁹ 1⁷2₁₁ → 3 ← 1₇2¹¹

(3)

Musical staff 3 consists of two staves. The top staff has a treble clef and a key signature of one sharp. The bottom staff has a treble clef and a key signature of one sharp. The notes are represented by dots on the staff. Below each staff is a series of numbered intervals and their inversions, indicating the horizontal movement between notes.

Top Staff Intervals:
 3₄ → 10.1 ← 3⁴ 3₁₀ → 12 ← 3¹⁰ 3₆ → 10.1 ← 3⁶ 3₁₂ → 12 ← 3¹² 3₈ → 10.1 ← 3⁸ 3₂ → 12 ← 3²

Bottom Staff Intervals:
 3₁ → 9.2 ← 3¹ 3₃ → 7 ← 3³ 3₅ → 9.2 ← 3⁵ 3₇ → 7 ← 3⁷ 3₉ → 9.2 ← 3⁹ 3₁₁ → 7 ← 3¹¹

(4)

Musical staff 4 consists of two staves. The top staff has a treble clef and a key signature of one flat. The bottom staff has a treble clef and a key signature of one sharp. The notes are represented by dots on the staff. Below each staff is a series of numbered intervals and their inversions, indicating the horizontal movement between notes.

Top Staff Intervals:
 4₄ → 6.2 ← 4⁴ 4₁₀ → 4 ← 4¹⁰ 4₆ → 6.2 ← 4⁶ 4₁₂ → 4 ← 4¹² 4₈ → 6.2 ← 4⁸ 4₂ → 4 ← 4²

Bottom Staff Intervals:
 4₁ → 1 ← 4¹ 4₃ → 5.1 ← 4³ 4₅ → 1 ← 4⁵ 4₇ → 5.1 ← 4⁷ 4₉ → 1 ← 4⁹ 4₁₁ → 5.1 ← 4¹¹

(5)

Musical staff 5 consists of two staves. The top staff has a treble clef and a key signature of one flat. The bottom staff has a treble clef and a key signature of one sharp. The notes are represented by dots on the staff. Below each staff is a series of numbered intervals and their inversions, indicating the horizontal movement between notes.

Top Staff Intervals:
 5₁₀6⁶ → 10.2 ← 5¹⁰6₆ 5₁₂6₁₀ → 8 ← 5¹²6¹⁰ 5₆6¹² → 10.2 ← 5⁶6₁₂ 5₂6₂ → 8 ← 5²6² 5₈6⁸ → 10.2 ← 5⁸6₈ 5₄6₄ → 8 ← 5⁴6⁴

Bottom Staff Intervals:
 5₁₆₁ → 9.1 ← 5¹⁶₁ 5₃6⁷ → 11 ← 5³6₇ 5₅6₅ → 9.1 ← 5⁵6⁵ 5₇6³ → 11 ← 5⁷6₃ 5₉6₉ → 9.1 ← 5⁹6⁹ 5₁₁6¹¹ → 11 ← 5¹¹6₁₁

(6)

Musical staff 6 consists of two staves. The top staff has a treble clef and a key signature of one flat. The bottom staff has a treble clef and a key signature of one sharp. The notes are represented by dots on the staff. Below each staff is a series of numbered intervals and their inversions, indicating the horizontal movement between notes.

Top Staff Intervals:
 5⁶6₁₂ → 10.2 ← 5₆6¹² 5₁₂6₁₀ → 8 ← 5¹²6¹⁰ 5¹⁰6₆ → 10.2 ← 5¹⁰6⁶ 5₂6₂ → 8 ← 5²6² 5⁸6₈ → 10.2 ← 5⁸6⁸ 5₄6₄ → 8 ← 5⁴6⁴

Bottom Staff Intervals:
 5₁₆₁ → 9.1 ← 5¹⁶₁ 5⁷6₃ → 11 ← 5⁷6³ 5₅6₅ → 9.1 ← 5⁵6⁵ 5³6₇ → 11 ← 5³6⁷ 5₉6₉ → 9.1 ← 5⁹6⁹ 5¹¹6₁₁ → 11 ← 5¹¹6₁₁

Horizontal Triads 2

(1)

Musical staff for exercise 1. It consists of two staves of five-line music. The notes are represented by dots and stems. Below each staff is a series of musical intervals and their corresponding numbers. The first staff starts with a dot at the top of the staff, followed by a stem pointing down, then another dot, and so on. The second staff starts with a dot at the bottom of the staff, followed by a stem pointing up, then another dot, and so on. The intervals are labeled below the staff.

Intervals for Staff 1:
 1⁴2² → 2 ← 1₄2₂ 1¹⁰2₄ → 6.1 ← 1₁₀2⁴ 1⁶2⁶ → 2 ← 1₆2₆ 1¹²2₁₂ → 6.1 ← 1₁₂2¹² 1⁸2⁸ → 2 ← 1₈2₈ 1²2₁₀ → 6.1 ← 1₂2¹⁰

Intervals for Staff 2:
 1¹²1 → 5.2 ← 1₁2₁ 1³2₃ → 3 ← 1₃2³ 1⁵2⁵ → 5.2 ← 1₅2₅ 1⁷2₁₁ → 3 ← 1₇2¹¹ 1⁹2⁹ → 5.2 ← 1₉2₉ 1¹¹2₇ → 3 ← 1₁₁2⁷

(2)

Musical staff for exercise 2. It consists of two staves of five-line music. The notes are represented by dots and stems. Below each staff is a series of musical intervals and their corresponding numbers. The first staff starts with a dot at the top of the staff, followed by a stem pointing down, then another dot, and so on. The second staff starts with a dot at the bottom of the staff, followed by a stem pointing up, then another dot, and so on. The intervals are labeled below the staff.

Intervals for Staff 1:
 1⁴2² → 2 ← 1₄2₂ 1₂2¹⁰ → 6.1 ← 1²2₁₀ 1⁶2⁶ → 2 ← 1₆2₆ 1₁₂2¹² → 6.1 ← 1¹²2₁₂ 1⁸2⁸ → 2 ← 1₈2₈ 1₁₀2⁴ → 6.1 ← 1¹⁰2₄

Intervals for Staff 2:
 1¹²1 → 5.2 ← 1₁2₁ 1₃2³ → 3 ← 1³2₃ 1⁵2⁵ → 5.2 ← 1₅2₅ 1₁₁2⁷ → 3 ← 1¹¹2₇ 1⁹2⁹ → 5.2 ← 1₉2₉ 1₇2¹¹ → 3 ← 1⁷2₁₁

(3)

Musical staff for exercise 3. It consists of two staves of five-line music. The notes are represented by dots and stems. Below each staff is a series of musical intervals and their corresponding numbers. The first staff starts with a dot at the top of the staff, followed by a stem pointing down, then another dot, and so on. The second staff starts with a dot at the bottom of the staff, followed by a stem pointing up, then another dot, and so on. The intervals are labeled below the staff.

Intervals for Staff 1:
 3⁴ → 10.1 ← 3₄ 3¹⁰ → 12 ← 3₁₀ 3⁶ → 10.1 ← 3₆ 3¹² → 12 ← 3₁₂ 3⁸ → 10.1 ← 3₈ 3² → 12 ← 3₂

Intervals for Staff 2:
 3¹ → 9.2 ← 3₁ 3³ → 7 ← 3₃ 3⁵ → 9.2 ← 3₅ 3⁷ → 7 ← 3₇ 3⁹ → 9.2 ← 3₉ 3¹¹ → 7 ← 3₁₁

(4)

Musical staff for exercise 4. It consists of two staves of five-line music. The notes are represented by dots and stems. Below each staff is a series of musical intervals and their corresponding numbers. The first staff starts with a dot at the top of the staff, followed by a stem pointing down, then another dot, and so on. The second staff starts with a dot at the bottom of the staff, followed by a stem pointing up, then another dot, and so on. The intervals are labeled below the staff.

Intervals for Staff 1:
 4⁴ → 6.2 ← 4₄ 4¹⁰ → 4 ← 4₁₀ 4⁶ → 6.2 ← 4₆ 4¹² → 4 ← 4₁₂ 4⁸ → 6.2 ← 4₈ 4² → 4 ← 4₂

Intervals for Staff 2:
 4¹ → 1 ← 4₁ 4³ → 5.1 ← 4₃ 4⁵ → 1 ← 4₅ 4⁷ → 5.1 ← 4₇ 4⁹ → 1 ← 4₉ 4¹¹ → 5.1 ← 4₁₁

(5)

Musical staff for exercise 5. It consists of two staves of five-line music. The notes are represented by dots and stems. Below each staff is a series of musical intervals and their corresponding numbers. The first staff starts with a dot at the top of the staff, followed by a stem pointing down, then another dot, and so on. The second staff starts with a dot at the bottom of the staff, followed by a stem pointing up, then another dot, and so on. The intervals are labeled below the staff.

Intervals for Staff 1:
 5¹⁰6₆ → 10.2 ← 5₁₀6⁶ 5¹²6¹⁰ → 8 ← 5₁₂6₁₀ 5⁶6₁₂ → 10.2 ← 5₆6¹² 5²6² → 8 ← 5₂6₂ 5⁸6₈ → 10.2 ← 5₈6⁸ 5⁴6⁴ → 8 ← 5₄6₄

Intervals for Staff 2:
 5¹6¹ → 9.1 ← 5₁6₁ 5³6₇ → 11 ← 5₃6⁷ 5⁵6⁵ → 9.1 ← 5₅6₅ 5⁷6₃ → 11 ← 5₇6³ 5⁹6⁹ → 9.1 ← 5₉6₉ 5¹¹6₁₁ → 11 ← 5₁₁6¹¹

(6)

Musical staff for exercise 6. It consists of two staves of five-line music. The notes are represented by dots and stems. Below each staff is a series of musical intervals and their corresponding numbers. The first staff starts with a dot at the top of the staff, followed by a stem pointing down, then another dot, and so on. The second staff starts with a dot at the bottom of the staff, followed by a stem pointing up, then another dot, and so on. The intervals are labeled below the staff.

Intervals for Staff 1:
 5₆6¹² → 10.2 ← 5⁶6₁₂ 5¹²6¹⁰ → 8 ← 5₁₂6₁₀ 5₁₀6⁶ → 10.2 ← 5¹⁰6₆ 5²6² → 8 ← 5₂6₂ 5₈6⁸ → 10.2 ← 5₈6⁸ 5⁴6⁴ → 8 ← 5₄6₄

Intervals for Staff 2:
 5¹6¹ → 9.1 ← 5₁6₁ 5₇6³ → 11 ← 5⁷6₃ 5⁵6⁵ → 9.1 ← 5₅6₅ 5₃6⁷ → 11 ← 5³6₇ 5⁹6⁹ → 9.1 ← 5₉6₉ 5₁₁6¹¹ → 11 ← 5₁₁6¹¹

6.10

Horizontal Equilateral Triangles 1

(1)

Musical staff 1 consists of two staves. The top staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. The bottom staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. Below each note is a label indicating its position: 1₂→, ←1², 1₄→, ←1⁴, 1₆→, ←1⁶, 1₈→, ←1⁸, 1₁₀→, ←1¹⁰, 1₁₂→, ←1¹².

Musical staff 2 consists of two staves. The top staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. The bottom staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. Below each note is a label indicating its position: 1₁₂₁→, 5.2, ←1¹²¹, 1₃₂³→, 3, ←1³₂₃, 1₅₂→, 5.2, ←1⁵₂⁵, 1₇₂¹¹→, 3, ←1⁷₂₁₁, 1₉₂→, 5.2, ←1⁹₂⁹, 1₁₁₂⁷→, 3, ←1¹¹₂₇.

(2)

Musical staff 3 consists of two staves. The top staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. The bottom staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. Below each note is a label indicating its position: 2₂→, ←2², 2₄→, ←2⁴, 2₆→, ←2⁶, 2₈→, ←2⁸, 2₁₀→, ←2¹⁰, 2₁₂→, ←2¹².

Musical staff 4 consists of two staves. The top staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. The bottom staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. Below each note is a label indicating its position: 1₁₂₁→, 5.2, ←1¹²¹, 1³₂₃→, 3, ←1₃₂³, 1₅₂→, 5.2, ←1⁵₂⁵, 1¹¹₂₇→, 3, ←1₁₁₂⁷, 1₉₂→, 5.2, ←1⁹₂⁹, 1⁷₂₁₁→, 3, ←1₇₂¹¹.

(3)

Musical staff 5 consists of two staves. The top staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. The bottom staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. Below each note is a label indicating its position: 3₂→, ←3², 3₄→, ←3⁴, 3₆→, ←3⁶, 3₈→, ←3⁸, 3₁₀→, ←3¹⁰, 3₁₂→, ←3¹².

Musical staff 6 consists of two staves. The top staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. The bottom staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. Below each note is a label indicating its position: 3₁→, 9.2, ←3¹, 3₃→, 7, ←3³, 3₅→, 9.2, ←3⁵, 3₇→, 7, ←3⁷, 3₉→, 9.2, ←3⁹, 3₁₁→, 7, ←3¹¹.

(4)

Musical staff 7 consists of two staves. The top staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. The bottom staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. Below each note is a label indicating its position: 4₂→, ←4², 4₄₆₁₂→, ←4⁴₆¹², 4₆→, ←4⁶, 4₈₆₄→, ←4⁸₆⁴, 4₁₀→, ←4¹⁰, 4₁₂₆₈→, ←4¹²₆⁸.

Musical staff 8 consists of two staves. The top staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. The bottom staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. Below each note is a label indicating its position: 4₁→, 1, ←4¹, 4₃→, 5.1, ←4³, 4₅→, 1, ←4⁵, 4₇→, 5.1, ←4⁷, 4₉→, 1, ←4⁹, 4₁₁→, 5.1, ←4¹¹.

(5)

Musical staff 9 consists of two staves. The top staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. The bottom staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. Below each note is a label indicating its position: 5₂→, ←5², 5₄→, ←5⁴, 5₆→, ←5⁶, 5₈→, ←5⁸, 5₁₀→, ←5¹⁰, 5₁₂→, ←5¹².

Musical staff 10 consists of two staves. The top staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. The bottom staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. Below each note is a label indicating its position: 5₁₆₁→, 9.1, ←5¹⁶¹, 5₃₆⁷→, 11, ←5³₆₇, 5₅₆₅→, 9.1, ←5⁵₆⁵, 5₇₆³→, 11, ←5⁷₆₃, 5₉₆₉→, 9.1, ←5⁹₆⁹, 5₁₁₆¹¹→, 11, ←5¹¹₆₁₁.

(6)

Musical staff 11 consists of two staves. The top staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. The bottom staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. Below each note is a label indicating its position: 6₂→, ←6², 4₈₆₄→, ←4⁸₆⁴, 6₆→, ←6⁶, 4₁₂₆₈→, ←4¹²₆⁸, 6₁₀→, ←6¹⁰, 4₄₆₁₂→, ←4⁴₆¹².

Musical staff 12 consists of two staves. The top staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. The bottom staff has notes on the 1st, 3rd, 5th, 7th, 9th, and 11th positions. Below each note is a label indicating its position: 5₁₆₁→, 9.1, ←5¹⁶¹, 5⁷₆₃→, 11, ←5⁷₆₃, 5₅₆₅→, 9.1, ←5⁵₆⁵, 5³₆₇→, 11, ←5³₆₇, 5₉₆₉→, 9.1, ←5⁹₆⁹, 5¹¹₆₁₁→, 11, ←5¹¹₆₁₁.

6.11

Horizontal Equilateral Triangles 2

(1)

1²→ ←1₂ 1⁴→ ←1₄ 1⁶→ ←1₆ 1⁸→ ←1₈ 1¹⁰→ ←1₁₀ 1¹²→ ←1₁₂

1¹²₁→ 5.2 ←1₁₂₁ 1³₂₃→ 3 ←1₃₂³ 1⁵₂₅→ 5.2 ←1₅₂₅ 1⁷₂₁₁→ 3 ←1₇₂¹¹ 1⁹₂₉→ 5.2 ←1₉₂₉ 1¹¹₂₇→ 3 ←1₁₁₂⁷

(2)

2²→ ←2₂ 2⁴→ ←2₄ 2⁶→ ←2₆ 2⁸→ ←2₈ 2¹⁰→ ←2₁₀ 2¹²→ ←2₁₂

1¹²₁→ 5.2 ←1₁₂₁ 1₃₂³→ 3 ←1³₂₃ 1⁵₂₅→ 5.2 ←1₅₂₅ 1₁₁₂⁷→ 3 ←1¹¹₂₇ 1⁹₂₉→ 5.2 ←1₉₂₉ 1₇₂¹¹→ 3 ←1⁷₂₁₁

(3)

3²→ ←3₂ 3⁴→ ←3₄ 3⁶→ ←3₆ 3⁸→ ←3₈ 3¹⁰→ ←3₁₀ 3¹²→ ←3₁₂

3¹→ 9.2 ←3₁ 3³→ 7 ←3₃ 3⁵→ 9.2 ←3₅ 3⁷→ 7 ←3₇ 3⁹→ 9.2 ←3₉ 3¹¹→ 7 ←3₁₁

(4)

4²→ ←4₂ 4⁴₆¹²→ ←4₆₁₂ 4⁶→ ←4₆ 4⁸₆⁴→ ←4₈₆₄ 4¹⁰→ ←4₁₀ 4¹²₆⁸→ ←4₁₂₆₈

4¹→ 1 ←4₁ 4³→ 5.1 ←4₃ 4⁵→ 1 ←4₅ 4⁷→ 5.1 ←4₇ 4⁹→ 1 ←4₉ 4¹¹→ 5.1 ←4₁₁

(5)

5²→ ←5₂ 5⁴→ ←5₄ 5⁶→ ←5₆ 5⁸→ ←5₈ 5¹⁰→ ←5₁₀ 5¹²→ ←5₁₂

5¹₆¹→ 9.1 ←5₁₆₁ 5³₆₇→ 11 ←5₃₆⁷ 5⁵₆⁵→ 9.1 ←5₅₆₅ 5⁷₆₃→ 11 ←5₇₆³ 5⁹₆₉→ 9.1 ←5₉₆₉ 5¹¹₆₁₁→ 11 ←5₁₁₆¹¹

(6)

6²→ ←6₂ 4⁸₆⁴→ ←4₈₆₄ 6⁶→ ←6₆ 4¹²₆₈→ ←4₁₂₆₈ 6¹⁰→ ←6₁₀ 4⁴₆¹²→ ←4₄₆₁₂

5¹₆¹→ 9.1 ←5₁₆₁ 5⁷₆₃→ 11 ←5⁷₆₃ 5⁵₆⁵→ 9.1 ←5₅₆₅ 5₃₆⁷→ 11 ←5³₆₇ 5⁹₆₉→ 9.1 ←5₉₆₉ 5₁₁₆₁₁→ 11 ←5¹¹₆₁₁

Outer Hexads 1
(Horizontal Outer Hexads in 6)

Outer Hexads 2
(Horizontal Outer Hexads in 6)

1

1₁|5 1₃|5 1₅|5 1₇|5 1₉|5 1₁₁|5

2

2₁|5 2₃|5 2₅|5 2₇|5 2₉|5 2₁₁|5

3

3₁|7 3₃|7 3₅|7 3₇|7 3₉|7 3₁₁|7

4

4₁|3 4₃|3 4₅|3 4₇|3 4₉|3 4₁₁|3

5

5₁|1 5₃|1 5₅|1 5₇|1 5₉|1 5₁₁|1

6

6₁|1 6₃|1 6₅|1 6₇|1 6₉|1 6₁₁|1

1

1¹|5 1³|5 1⁵|5 1⁷|5 1⁹|5 1¹¹|5

2

2¹|5 2³|5 2⁵|5 2⁷|5 2⁹|5 2¹¹|5

3

3¹|7 3³|7 3⁵|7 3⁷|7 3⁹|7 3¹¹|7

4

4¹|3 4³|3 4⁵|3 4⁷|3 4⁹|3 4¹¹|3

5

5¹|1 5³|1 5⁵|1 5⁷|1 5⁹|1 5¹¹|1

6

6¹|1 6³|1 6⁵|1 6⁷|1 6⁹|1 6¹¹|1

Outer Hexads 3

(\Leftrightarrow)

Outer Hexads 4

①

②

③

④

⑤

⑥

①

②

③

④

⑤

⑥

Inner Hexads 1

(Horizontal Inner Hexads in 6)

(1)

$b\flat$

$1_2|4 \quad 1_4|4 \quad 1_6|4 \quad 1_8|4 \quad 1_{10}|4 \quad 1_{12}|4$

$2_2|4 \quad 2_4|4 \quad 2_6|4 \quad 2_8|4 \quad 2_{10}|4 \quad 2_{12}|4$

$e\flat$

$3_2|2 \quad 3_4|2 \quad 3_6|2 \quad 3_8|2 \quad 3_{10}|2 \quad 3_{12}|2$

bb

$4_2|6 \quad 4_4|6 \quad 4_6|6 \quad 4_8|6 \quad 4_{10}|6 \quad 4_{12}|6$

eb

$5_2|8 \quad 5_4|8 \quad 5_6|8 \quad 5_8|8 \quad 5_{10}|8 \quad 5_{12}|8$

bb

$6_2|8 \quad 6_4|8 \quad 6_6|8 \quad 6_8|8 \quad 6_{10}|8 \quad 6_{12}|8$

(⇒)

Inner Hexads 2

(Horizontal Inner Hexads in 6)

(1)

d

$1^2|4 \quad 1^4|4 \quad 1^6|4 \quad 1^8|4 \quad 1^c \quad 1^{bb}$

$2^2|4 \quad 2^4|4 \quad 2^6|4 \quad 2^8|4 \quad 2^{10}|4 \quad 2^{12}|4$

$c\sharp$

$3^2|2 \quad 3^4|2 \quad 3^6|2 \quad 3^8|2 \quad 3^{10}|2 \quad 3^{12}|2$

c

$4^2|6 \quad 4^4|6 \quad 4^6|6 \quad 4^8|6 \quad 4^{10}|6 \quad 4^{12}|6$

c

$5^2|8 \quad 5^4|8 \quad 5^6|8 \quad 5^8|8 \quad 5^{10}|8 \quad 5^{12}|8$

c

$6^2|8 \quad 6^4|8 \quad 6^6|8 \quad 6^8|8 \quad 6^{10}|8 \quad 6^{12}|8$

Inner Hexads 3

(\Leftrightarrow)

Inner Hexads 4

1

$1_2|4 \quad 1^4|4 \quad 1_6|4 \quad 1^8|4 \quad 1_{10}|4 \quad 1^{12}|4$

2

$2_2|4 \quad 2^4|4 \quad 2_6|4 \quad 2^8|4 \quad 2_{10}|4 \quad 2^{12}|4$

3

$3_2|2 \quad 3^4|2 \quad 3_6|2 \quad 3^8|2 \quad 3_{10}|2 \quad 3^{12}|2$

4

$4_2|6 \quad 4^4|6 \quad 4_6|6 \quad 4^8|6 \quad 4_{10}|6 \quad 4^{12}|6$

5

$5_2|8 \quad 5^4|8 \quad 5_6|8 \quad 5^8|8 \quad 5_{10}|8 \quad 5^{12}|8$

6

$6_2|8 \quad 6^4|8 \quad 6_6|8 \quad 6^8|8 \quad 6_{10}|8 \quad 6^{12}|8$

1

$1^2|4 \quad 1_4|4 \quad 1^6|4 \quad 1_8|4 \quad 1^{10}|4 \quad 1_{12}|4$

2

$2^2|4 \quad 2_4|4 \quad 2^6|4 \quad 2_8|4 \quad 2^{10}|4 \quad 2_{12}|4$

3

$3^2|2 \quad 3_4|2 \quad 3^6|2 \quad 3_8|2 \quad 3^{10}|2 \quad 3_{12}|2$

4

$4^2|6 \quad 4_4|6 \quad 4^6|6 \quad 4_8|6 \quad 4^{10}|6 \quad 4_{12}|6$

5

$5^2|8 \quad 5_4|8 \quad 5^6|8 \quad 5_8|8 \quad 5^{10}|8 \quad 5_{12}|8$

6

$6^2|8 \quad 6_4|8 \quad 6^6|8 \quad 6_8|8 \quad 6^{10}|8 \quad 6_{12}|8$

Outer and Inner Hexads 1

(Horizontal Dodecads in 6)

1

$1_1 \quad 1_2 \quad 1_3 \quad 1_4 \quad 1_5 \quad 1_6 \quad 1_7 \quad 1_8 \quad 1_9 \quad 1_{10} \quad 1_{11} \quad 1_{12}$

$5 \quad 4 \quad 5 \quad 4$

2

$2_1 \quad 2_2 \quad 2_3 \quad 2_4 \quad 2_5 \quad 2_6 \quad 2_7 \quad 2_8 \quad 2_9 \quad 2_{10} \quad 2_{11} \quad 2_{12}$

$5 \quad 4 \quad 5 \quad 4$

3

$3_1 \quad 3_2 \quad 3_3 \quad 3_4 \quad 3_5 \quad 3_6 \quad 3_7 \quad 3_8 \quad 3_9 \quad 3_{10} \quad 3_{11} \quad 3_{12}$

$7 \quad 2 \quad 7 \quad 2$

4

$4_1 \quad 4_2 \quad 4_3 \quad 4_4 \quad 4_5 \quad 4_6 \quad 4_7 \quad 4_8 \quad 4_9 \quad 4_{10} \quad 4_{11} \quad 4_{12}$

$3 \quad 6 \quad 3 \quad 6$

5

$5_1 \quad 5_2 \quad 5_3 \quad 5_4 \quad 5_5 \quad 5_6 \quad 5_7 \quad 5_8 \quad 5_9 \quad 5_{10} \quad 5_{11} \quad 5_{12}$

$1 \quad 8 \quad 1 \quad 8$

6

$6_1 \quad 6_2 \quad 6_3 \quad 6_4 \quad 6_5 \quad 6_6 \quad 6_7 \quad 6_8 \quad 6_9 \quad 6_{10} \quad 6_{11} \quad 6_{12}$

$1 \quad 8 \quad 1 \quad 8$

7.5

Outer and Inner Hexads 2

(Horizontal Dodecads in 6)

(1)

1¹ 1² 1³ 1⁴ 1⁵ 1⁶ 1⁷ 1⁸ 1⁹ 1¹⁰ 1¹¹ 1¹²
5 4 5 4 5 4 5 4 5 4 5 4

(2)

2¹ 2² 2³ 2⁴ 2⁵ 2⁶ 2⁷ 2⁸ 2⁹ 2¹⁰ 2¹¹ 2¹²
5 4 5 4 5 4 5 4 5 4 5 4

(3)

3¹ 3² 3³ 3⁴ 3⁵ 3⁶ 3⁷ 3⁸ 3⁹ 3¹⁰ 3¹¹ 3¹²
7 2 7 2 7 2 7 2 7 2 7 2

(4)

4¹ 4² 4³ 4⁴ 4⁵ 4⁶ 4⁷ 4⁸ 4⁹ 4¹⁰ 4¹¹ 4¹²
3 6 3 6 3 6 3 6 3 6 3 6

(5)

5¹ 5² 5³ 5⁴ 5⁵ 5⁶ 5⁷ 5⁸ 5⁹ 5¹⁰ 5¹¹ 5¹²
1 8 1 8 1 8 1 8 1 8 1 8

(6)

6¹ 6² 6³ 6⁴ 6⁵ 6⁶ 6⁷ 6⁸ 6⁹ 6¹⁰ 6¹¹ 6¹²
1 8 1 8 1 8 1 8 1 8 1 8

Outer and Inner Hexads Expanded 1...

(Horizontal Dodecads in 6)

(1)

Musical staff showing 12 notes labeled 1₁ through 1₁₂. The notes are represented by vertical stems with dots at different heights. The staff has a treble clef and a bass clef.

Label	1 ₁	1 ₂	1 ₃	1 ₄	1 ₅	1 ₆	1 ₇	1 ₈	1 ₉	1 ₁₀	1 ₁₁	1 ₁₂
Number	5	4	5	4	5	4	5	4	5	4	5	4

(2)

Musical staff showing 12 notes labeled 2₁ through 2₁₂. The notes are represented by vertical stems with dots at different heights. The staff has a treble clef and a bass clef.

Label	2 ₁	2 ₂	2 ₃	2 ₄	2 ₅	2 ₆	2 ₇	2 ₈	2 ₉	2 ₁₀	2 ₁₁	2 ₁₂
Number	5	4	5	4	5	4	5	4	5	4	5	4

(3)

Musical staff showing 12 notes labeled 3₁ through 3₁₂. The notes are represented by vertical stems with dots at different heights. The staff has a treble clef and a bass clef.

Label	3 ₁	3 ₂	3 ₃	3 ₄	3 ₅	3 ₆	3 ₇	3 ₈	3 ₉	3 ₁₀	3 ₁₁	3 ₁₂
Number	7	2	7	2	7	2	7	2	7	2	7	2

...Outer and Inner Hexads Expanded 1
 (Horizontal Dodecads in 6)

(4)

4₁
3

4₂
6

4₃
3

4₄
6

4₅
3

4₆
6

4₇
3

4₈
6

4₉
3

4₁₀
6

4₁₁
3

4₁₂
6

(5)

5₁
1

5₂
8

5₃
1

5₄
8

5₅
1

5₆
8

5₇
1

5₈
8

5₉
1

5₁₀
8

5₁₁
1

5₁₂
8

(6)

6₁
1

6₂
8

6₃
1

6₄
8

6₅
1

6₆
8

6₇
1

6₈
8

6₉
1

6₁₀
8

6₁₁
1

6₁₂
8

Outer and Inner Hexads Expanded 2...

(Horizontal Dodecads in 6)

(1)

Musical staff showing 12 notes in a hexad pattern. The notes are represented by vertical stems with dots at the top. The first note is labeled 'd' with a breve stem. The second note is labeled 'eb' with a breve stem. The third note is labeled 'd' with a breve stem. The fourth note is labeled 'eb' with a breve stem. The fifth note is labeled 'c' with a breve stem. The sixth note is labeled 'c#' with a breve stem. The seventh note is labeled 'bb' with a breve stem. The eighth note is labeled 'b' with a breve stem. The ninth note is labeled 'b' with a breve stem. The tenth note is labeled 'bb' with a breve stem. The eleventh note is labeled 'b' with a breve stem. The twelfth note is labeled 'b' with a breve stem.

1^1	1^2	1^3	1^4	1^5	1^6	1^7	1^8	1^9	1^{10}	1^{11}	1^{12}
5	4	5	4	5	4	5	4	5	4	5	4

(2)

Musical staff showing 12 notes in a hexad pattern. The notes are represented by vertical stems with dots at the top. The first note is labeled 'eb' with a breve stem. The second note is labeled 'd' with a breve stem. The third note is labeled 'd' with a breve stem. The fourth note is labeled 'eb' with a breve stem. The fifth note is labeled 'c' with a breve stem. The sixth note is labeled 'c#' with a breve stem. The seventh note is labeled 'b' with a breve stem. The eighth note is labeled 'bb' with a breve stem. The ninth note is labeled 'b' with a breve stem. The tenth note is labeled 'bb' with a breve stem. The eleventh note is labeled 'b' with a breve stem. The twelfth note is labeled 'b' with a breve stem.

2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}	2^{11}	2^{12}
5	4	5	4	5	4	5	4	5	4	5	4

(3)

Musical staff showing 12 notes in a hexad pattern. The notes are represented by vertical stems with dots at the top. The first note is labeled 'c#' with a breve stem. The second note is labeled 'd#' with a breve stem. The third note is labeled 'b' with a breve stem. The fourth note is labeled 'c' with a breve stem. The fifth note is labeled 'd' with a breve stem. The sixth note is labeled 'bb' with a breve stem. The seventh note is labeled 'b' with a breve stem. The eighth note is labeled 'b' with a breve stem. The ninth note is labeled 'bb' with a breve stem. The tenth note is labeled 'b' with a breve stem. The eleventh note is labeled 'b' with a breve stem. The twelfth note is labeled 'b' with a breve stem.

3^1	3^2	3^3	3^4	3^5	3^6	3^7	3^8	3^9	3^{10}	3^{11}	3^{12}
7	2	7	2	7	2	7	2	7	2	7	2

...Outer and Inner Hexads Expanded 2
 (Horizontal Dodecads in 6)

(4)

4 ¹	4 ²	4 ³	4 ⁴	4 ⁵	4 ⁶	4 ⁷	4 ⁸	4 ⁹	4 ¹⁰	4 ¹¹	4 ¹²
3	6	3	6	3	6	3	6	3	6	3	6

(5)

5 ¹	5 ²	5 ³	5 ⁴	5 ⁵	5 ⁶	5 ⁷	5 ⁸	5 ⁹	5 ¹⁰	5 ¹¹	5 ¹²
1	8	1	8	1	8	1	8	1	8	1	8

(6)

6 ¹	6 ²	6 ³	6 ⁴	6 ⁵	6 ⁶	6 ⁷	6 ⁸	6 ⁹	6 ¹⁰	6 ¹¹	6 ¹²
1	8	1	8	1	8	1	8	1	8	1	8

Hexads Sorted by Rotations 1...

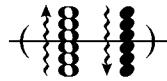


Diagram illustrating hexads sorted by rotations, presented in three sections (I, II, III) across six staves. Each staff contains two hexads, each with six possible rotations. The hexads are represented by vertical columns of six notes on a five-line staff.

Section I:

- 1₁|5 1₂|4
- 2₁|5 2₂|4
- 3₁|7 3₂|2
- 4₁|3 4₂|6
- 5₁|1 5₂|8
- 6₁|1 6₂|8

Section II:

- 1₃|5 1₄|4
- 2₃|5 2₄|4
- 3₃|7 3₄|2
- 4₃|3 4₄|6
- 5₃|1 5₄|8
- 6₃|1 6₄|8

Section III:

- 1₅|5 1₆|4
- 2₅|5 2₆|4 b
- 3₅|7 3₆|2
- 4₅|3 4₆|6
- 5₅|1 5₆|8
- 6₅|1 6₆|8

Annotations include Roman numerals I, II, III, and circled numbers 1 through 6, along with various letter and symbol labels such as b, bb, #, d, db, eb, c, and d#.

...Hexads Sorted by Rotations 1

IV

17|5 18|4 27|5 28|4 37|7 38|2

1 2 3

5

47|3 48|6 57|1 58|8 67|1 68|8

4 5 6

V

19|5 110|4 29|5 210|4 39|7 310|2

1 2 3

6

49|3 410|6 59|1 510|8 69|1 610|8

4 5 6

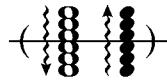
VI

11|5 112|4 211|5 212|4 311|7 312|2

c# b d
bb c# bb
b eb bb
bb ebb b

4 5 6

Hexads Sorted by Rotations 2...



I II III

...Hexads Sorted by Rotations 2

IV

17|5 18|4

27|5 28|4

37|7 38|2

d

47|3 48|6

57|1 58|8

67|1 68|8

V

19|5 10|4

29|5 210|4

39|7 310|2

b

49|3 410|6

59|1 510|8

69|1 610|8

VI

111|5 112|4

211|5 212|4

311|7 312|2

db

411|3 412|6

511|1 512|8

611|1 612|8

Reversible Outer Hexads 1

(1)

(2)

1₁\1¹ 1₃\1³ 1₅\1⁵ 1₇\1⁷ 1₉\1⁹ 1₁₁\1¹¹
5 5 5 5 5 5

2₁\2¹ 2₃\2³ 2₅\2⁵ 2₇\2⁷ 2₉\2⁹ 2₁₁\2¹¹
5 5 5 5 5 5

(3)

(4)

3₁\3¹ 3₃\3³ 3₅\3⁵ 3₇\3⁷ 3₉\3⁹ 3₁₁\3¹¹
7 7 7 7 7 7

4₁\4¹ 4₃\4³ 4₅\4⁵ 4₇\4⁷ 4₉\4⁹ 4₁₁\4¹¹
3 3 3 3 3 3

(5)

(6)

5₁\5¹ 5₃\5³ 5₅\5⁵ 5₇\5⁷ 5₉\5⁹ 5₁₁\5¹¹
1 1 1 1 1 1

6₁\6¹ 6₃\6³ 6₅\6⁵ 6₇\6⁷ 6₉\6⁹ 6₁₁\6¹¹
1 1 1 1 1 1

Reversible Outer Hexads 2

(1)

$1^1\backslash 1_1$
 5
 $1^3\backslash 1_3$
 5
 $1^5\backslash 1_5$
 5
 $1^7\backslash 1_7$
 5
 $1^9\backslash 1_9$
 5
 $1^{11}\backslash 1_{11}$
 5

(2)

$2^1\backslash 2_1$
 5
 $2^3\backslash 2_3$
 5
 $2^5\backslash 2_5$
 5
 $2^7\backslash 2_7$
 5
 $2^9\backslash 2_9$
 5
 $2^{11}\backslash 2_{11}$
 5

(3)

$3^1\backslash 3_1$
 7
 $3^3\backslash 3_3$
 7
 $3^5\backslash 3_5$
 7
 $3^7\backslash 3_7$
 7
 $3^9\backslash 3_9$
 7
 $3^{11}\backslash 3_{11}$
 7

(4)

$4^1\backslash 4_1$
 3
 $4^3\backslash 4_3$
 3
 $4^5\backslash 4_5$
 3
 $4^7\backslash 4_7$
 3
 $4^9\backslash 4_9$
 3
 $4^{11}\backslash 4_{11}$
 3

(5)

$5^1\backslash 5_1$
 1
 $5^3\backslash 5_3$
 1
 $5^5\backslash 5_5$
 1
 $5^7\backslash 5_7$
 1
 $5^9\backslash 5_9$
 1
 $5^{11}\backslash 5_{11}$
 1

(6)

$6^1\backslash 6_1$
 1
 $6^3\backslash 6_3$
 1
 $6^5\backslash 6_5$
 1
 $6^7\backslash 6_7$
 1
 $6^9\backslash 6_9$
 1
 $6^{11}\backslash 6_{11}$
 1

* = reverse point.

Reversible Inner Hexads 1

(1)

$1_1 \backslash 1^1$ $1_3 \backslash 1^3$ $1_5 \backslash 1^5$ $1_7 \backslash 1^7$ $1_9 \backslash 1^9$ $1_{11} \backslash 1^{11}$
 4 4 4 4 4 4

(2)

$2_1 \backslash 2^1$ $2_3 \backslash 2^3$ $2_5 \backslash 2^5$ $2_7 \backslash 2^7$ $2_9 \backslash 2^9$ $2_{11} \backslash 2^{11}$
 4 4 4 4 4 4

(3)

$3_1 \backslash 3^1$ $3_3 \backslash 3^3$ $3_5 \backslash 3^5$ $3_7 \backslash 3^7$ $3_9 \backslash 3^9$ $3_{11} \backslash 3^{11}$
 2 2 2 2 2 2

(4)

$4_1 \backslash 4^1$ $4_3 \backslash 4^3$ $4_5 \backslash 4^5$ $4_7 \backslash 4^7$ $4_9 \backslash 4^9$ $4_{11} \backslash 4^{11}$
 6 6 6 6 6 6

(5)

$5_1 \backslash 5^1$ $5_3 \backslash 5^3$ $5_5 \backslash 5^5$ $5_7 \backslash 5^7$ $5_9 \backslash 5^9$ $5_{11} \backslash 5^{11}$
 8 8 8 8 8 8

(6)

$6_1 \backslash 6^1$ $6_3 \backslash 6^3$ $6_5 \backslash 6^5$ $6_7 \backslash 6^7$ $6_9 \backslash 6^9$ $6_{11} \backslash 6^{11}$
 8 8 8 8 8 8

* = reverse point.

Reversible Inner Hexads 2

(1)

Musical staff showing hexad 1 in various inversions. The staff has 8 measures. Measures 1-4 show inversions of hexad 1. Measures 5-8 show inversions of hexad 11. The staff has a bass clef, a key signature of one sharp, and a common time signature.

$1^1\backslash 1_1$
 4
 $1^3\backslash 1_3$
 4
 $1^5\backslash 1_5$
 4
 $1^7\backslash 1_7$
 4
 $1^9\backslash 1_9$
 4
 $1^{11}\backslash 1_{11}$
 4

(2)

Musical staff showing hexad 2 in various inversions. The staff has 8 measures. Measures 1-4 show inversions of hexad 2. Measures 5-8 show inversions of hexad 11. The staff has a bass clef, a key signature of one sharp, and a common time signature.

$2^1\backslash 2_1$
 4
 $2^3\backslash 2_3$
 4
 $2^5\backslash 2_5$
 4
 $2^7\backslash 2_7$
 4
 $2^9\backslash 2_9$
 4
 $2^{11}\backslash 2_{11}$
 4

(3)

Musical staff showing hexad 3 in various inversions. The staff has 8 measures. Measures 1-4 show inversions of hexad 3. Measures 5-8 show inversions of hexad 11. The staff has a bass clef, a key signature of one sharp, and a common time signature.

$3^1\backslash 3_1$
 2
 $3^3\backslash 3_3$
 2
 $3^5\backslash 3_5$
 2
 $3^7\backslash 3_7$
 2
 $3^9\backslash 3_9$
 2
 $3^{11}\backslash 3_{11}$
 2

(4)

Musical staff showing hexad 4 in various inversions. The staff has 8 measures. Measures 1-4 show inversions of hexad 4. Measures 5-8 show inversions of hexad 11. The staff has a bass clef, a key signature of one sharp, and a common time signature.

$4^1\backslash 4_1$
 6
 $4^3\backslash 4_3$
 6
 $4^5\backslash 4_5$
 6
 $4^7\backslash 4_7$
 6
 $4^9\backslash 4_9$
 6
 $4^{11}\backslash 4_{11}$
 6

(5)

Musical staff showing hexad 5 in various inversions. The staff has 8 measures. Measures 1-4 show inversions of hexad 5. Measures 5-8 show inversions of hexad 11. The staff has a bass clef, a key signature of one sharp, and a common time signature.

$5^1\backslash 5_1$
 8
 $5^3\backslash 5_3$
 8
 $5^5\backslash 5_5$
 8
 $5^7\backslash 5_7$
 8
 $5^9\backslash 5_9$
 8
 $5^{11}\backslash 5_{11}$
 8

(6)

Musical staff showing hexad 6 in various inversions. The staff has 8 measures. Measures 1-4 show inversions of hexad 6. Measures 5-8 show inversions of hexad 11. The staff has a bass clef, a key signature of one sharp, and a common time signature.

$6^1\backslash 6_1$
 8
 $6^3\backslash 6_3$
 8
 $6^5\backslash 6_5$
 8
 $6^7\backslash 6_7$
 8
 $6^9\backslash 6_9$
 8
 $6^{11}\backslash 6_{11}$
 8

* = reverse point.

Horizontal Hexads 1

(1)

4 $1_2 \rightarrow$
 $\leftarrow 1^4$

5 $1_1 \rightarrow$
 $\leftarrow 1^3$

4 $1_4 \rightarrow$
 $\leftarrow 1^6$

5 $1_3 \rightarrow$
 $\leftarrow 1^5$

4 $1_6 \rightarrow$
 $\leftarrow 1^8$

5 $1_5 \rightarrow$
 $\leftarrow 1^7$

4 $1_8 \rightarrow$
 $\leftarrow 1^{10}$

5 $1_7 \rightarrow$
 $\leftarrow 1^9$

4 $1_{10} \rightarrow$
 $\leftarrow 1^{12}$

5 $1_{11} \rightarrow$
 $\leftarrow 1^1$

4 $1_{12} \rightarrow$
 $\leftarrow 1^2$

(2)

4 $2_2 \rightarrow$
 $\leftarrow 2^4$

5 $2_1 \rightarrow$
 $\leftarrow 2^3$

4 $2_4 \rightarrow$
 $\leftarrow 2^6$

5 $2_3 \rightarrow$
 $\leftarrow 2^5$

4 $2_6 \rightarrow$
 $\leftarrow 2^8$

5 $2_5 \rightarrow$
 $\leftarrow 2^7$

4 $2_8 \rightarrow$
 $\leftarrow 2^{10}$

5 $2_7 \rightarrow$
 $\leftarrow 2^9$

4 $2_{10} \rightarrow$
 $\leftarrow 2^{12}$

5 $2_9 \rightarrow$
 $\leftarrow 2^{11}$

4 $2_{12} \rightarrow$
 $\leftarrow 2^2$

5 $2_{11} \rightarrow$
 $\leftarrow 2^1$

(3)

2 $3_2 \rightarrow$
 $\leftarrow 3^4$

7 $3_1 \rightarrow$
 $\leftarrow 3^3$

2 $3_4 \rightarrow$
 $\leftarrow 3^6$

7 $3_3 \rightarrow$
 $\leftarrow 3^5$

2 $3_6 \rightarrow$
 $\leftarrow 3^8$

7 $3_5 \rightarrow$
 $\leftarrow 3^7$

2 $3_8 \rightarrow$
 $\leftarrow 3^{10}$

7 $3_7 \rightarrow$
 $\leftarrow 3^9$

2 $3_{10} \rightarrow$
 $\leftarrow 3^{12}$

7 $3_9 \rightarrow$
 $\leftarrow 3^{11}$

2 $3_{12} \rightarrow$
 $\leftarrow 3^2$

7 $3_{11} \rightarrow$
 $\leftarrow 3^1$

(4)

6 $4_2 \rightarrow$
 $\leftarrow 4^4$

3 $4_1 \rightarrow$
 $\leftarrow 4^3$

6 $4_4 \rightarrow$
 $\leftarrow 4^6$

3 $4_3 \rightarrow$
 $\leftarrow 4^5$

6 $4_6 \rightarrow$
 $\leftarrow 4^8$

3 $4_5 \rightarrow$
 $\leftarrow 4^7$

6 $4_8 \rightarrow$
 $\leftarrow 4^{10}$

3 $4_7 \rightarrow$
 $\leftarrow 4^9$

6 $4_{10} \rightarrow$
 $\leftarrow 4^{12}$

3 $4_9 \rightarrow$
 $\leftarrow 4^{11}$

6 $4_{12} \rightarrow$
 $\leftarrow 4^2$

3 $4_{11} \rightarrow$
 $\leftarrow 4^1$

(5)

8 $5_2 \rightarrow$
 $\leftarrow 5^4$

1 $5_1 \rightarrow$
 $\leftarrow 5^3$

8 $5_4 \rightarrow$
 $\leftarrow 5^6$

1 $5_3 \rightarrow$
 $\leftarrow 5^5$

8 $5_6 \rightarrow$
 $\leftarrow 5^8$

1 $5_5 \rightarrow$
 $\leftarrow 5^7$

8 $5_8 \rightarrow$
 $\leftarrow 5^{10}$

1 $5_7 \rightarrow$
 $\leftarrow 5^9$

8 $5_{10} \rightarrow$
 $\leftarrow 5^{12}$

1 $5_9 \rightarrow$
 $\leftarrow 5^{11}$

8 $5_{12} \rightarrow$
 $\leftarrow 5^2$

1 $5_{11} \rightarrow$
 $\leftarrow 5^1$

(6)

8 $6_2 \rightarrow$
 $\leftarrow 6^4$

1 $6_1 \rightarrow$
 $\leftarrow 6^3$

8 $6_4 \rightarrow$
 $\leftarrow 6^6$

1 $6_3 \rightarrow$
 $\leftarrow 6^5$

8 $6_6 \rightarrow$
 $\leftarrow 6^8$

1 $6_5 \rightarrow$
 $\leftarrow 6^7$

8 $6_8 \rightarrow$
 $\leftarrow 6^{10}$

1 $6_7 \rightarrow$
 $\leftarrow 6^9$

8 $6_{10} \rightarrow$
 $\leftarrow 6^{12}$

1 $6_9 \rightarrow$
 $\leftarrow 6^{11}$

8 $6_{12} \rightarrow$
 $\leftarrow 6^2$

1 $6_{11} \rightarrow$
 $\leftarrow 6^1$

Horizontal Hexads 2

(1)

4 $\leftarrow 1_{12}$ $\leftarrow 1_{10}$ $\leftarrow 1_8$ $\leftarrow 1_6$ $\leftarrow 1_4$ $\leftarrow 1_2$ 4
 $1^2 \rightarrow$ $1^{12} \rightarrow$ $1^{10} \rightarrow$ $1^8 \rightarrow$ $1^6 \rightarrow$ $1^4 \rightarrow$
5 $\leftarrow 1_{11}$ $\leftarrow 1_9$ $\leftarrow 1_7$ $\leftarrow 1_5$ $\leftarrow 1_3$ $\leftarrow 1_1$ 5
 $1^1 \rightarrow$ $1^{11} \rightarrow$ $1^9 \rightarrow$ $1^7 \rightarrow$ $1^5 \rightarrow$ $1^3 \rightarrow$

(2)

4 $\leftarrow 2_{12}$ $\leftarrow 2_{10}$ $\leftarrow 2_8$ $\leftarrow 2_6$ $\leftarrow 2_4$ $\leftarrow 2_2$ 4
 $2^2 \rightarrow$ $2^{12} \rightarrow$ $2^{10} \rightarrow$ $2^8 \rightarrow$ $2^6 \rightarrow$ $2^4 \rightarrow$
5 $\leftarrow 2_{11}$ $\leftarrow 2_9$ $\leftarrow 2_7$ $\leftarrow 2_5$ $\leftarrow 2_3$ $\leftarrow 2_1$ 5
 $2^1 \rightarrow$ $2^{11} \rightarrow$ $2^9 \rightarrow$ $2^7 \rightarrow$ $2^5 \rightarrow$ $2^3 \rightarrow$

(3)

2 $\leftarrow 3_{12}$ $\leftarrow 3_{10}$ $\leftarrow 3_8$ $\leftarrow 3_6$ $\leftarrow 3_4$ $\leftarrow 3_2$ 2
 $3^2 \rightarrow$ $3^{12} \rightarrow$ $3^{10} \rightarrow$ $3^8 \rightarrow$ $3^6 \rightarrow$ $3^4 \rightarrow$
7 $\leftarrow 3_{11}$ $\leftarrow 3_9$ $\leftarrow 3_7$ $\leftarrow 3_5$ $\leftarrow 3_3$ $\leftarrow 3_1$ 7
 $3^1 \rightarrow$ $3^{11} \rightarrow$ $3^9 \rightarrow$ $3^7 \rightarrow$ $3^5 \rightarrow$ $3^3 \rightarrow$

(4)

6 $\leftarrow 4_{12}$ $\leftarrow 4_{10}$ $\leftarrow 4_8$ $\leftarrow 4_6$ $\leftarrow 4_4$ $\leftarrow 4_2$ 6
 $4^2 \rightarrow$ $4^{12} \rightarrow$ $4^{10} \rightarrow$ $4^8 \rightarrow$ $4^6 \rightarrow$ $4^4 \rightarrow$
3 $\leftarrow 4_{11}$ $\leftarrow 4_9$ $\leftarrow 4_7$ $\leftarrow 4_5$ $\leftarrow 4_3$ $\leftarrow 4_1$ 3
 $4^1 \rightarrow$ $4^{11} \rightarrow$ $4^9 \rightarrow$ $4^7 \rightarrow$ $4^5 \rightarrow$ $4^3 \rightarrow$

(5)

8 $\leftarrow 5_{12}$ $\leftarrow 5_{10}$ $\leftarrow 5_8$ $\leftarrow 5_6$ $\leftarrow 5_4$ $\leftarrow 5_2$ 8
 $5^2 \rightarrow$ $5^{12} \rightarrow$ $5^{10} \rightarrow$ $5^8 \rightarrow$ $5^6 \rightarrow$ $5^4 \rightarrow$
1 $\leftarrow 5_{11}$ $\leftarrow 5_9$ $\leftarrow 5_7$ $\leftarrow 5_5$ $\leftarrow 5_3$ $\leftarrow 5_1$ 1
 $5^1 \rightarrow$ $5^{11} \rightarrow$ $5^9 \rightarrow$ $5^7 \rightarrow$ $5^5 \rightarrow$ $5^3 \rightarrow$

(6)

8 $\leftarrow 6_{12}$ $\leftarrow 6_{10}$ $\leftarrow 6_8$ $\leftarrow 6_6$ $\leftarrow 6_4$ $\leftarrow 6_2$ 8
 $6^2 \rightarrow$ $6^{12} \rightarrow$ $6^{10} \rightarrow$ $6^8 \rightarrow$ $6^6 \rightarrow$ $6^4 \rightarrow$
1 $\leftarrow 6_{11}$ $\leftarrow 6_9$ $\leftarrow 6_7$ $\leftarrow 6_5$ $\leftarrow 6_3$ $\leftarrow 6_1$ 1
 $6^1 \rightarrow$ $6^{11} \rightarrow$ $6^9 \rightarrow$ $6^7 \rightarrow$ $6^5 \rightarrow$ $6^3 \rightarrow$

Enneads...

(1)

Musical score for Ennead 1, featuring three staves (Treble, Bass, and Bass) over six measures. Measure 1 consists of two half-measures: 1₁ (2 notes) and 1¹ (2 notes). Measures 2 through 6 each consist of a half-measure (2 notes) followed by a dotted half-measure (3.2 notes). The music is in common time, with a key signature of one sharp (F# major). Measures 1₁ through 1⁴ are in F# major, while 1₅ through 1⁶ switch to C major.

(2)

Musical score for Ennead 2, featuring three staves (Treble, Bass, and Bass) over six measures. Measure 1 consists of two half-measures: 2₁ (2 notes) and 2¹ (2 notes). Measures 2 through 6 each consist of a half-measure (2 notes) followed by a dotted half-measure (3.2 notes). The music is in common time, with a key signature of one sharp (F# major).

(3)

Musical score for Ennead 3, featuring three staves (Treble, Bass, and Bass) over six measures. Measure 1 consists of two half-measures: 3₁ (4 notes) and 3¹ (4 notes). Measures 2 through 6 each consist of a half-measure (4 notes) followed by a dotted half-measure (5.2 notes). The music is in common time, with a key signature of one sharp (F# major).

...Enneads...

(1)

Musical score for Ennead 1, 15 measures. The score consists of four staves of music. Measure numbers 1₇ through 1₁₂ are indicated below the staff. Measure 1₉ contains a note labeled "c♯". Measure 1₁₀ contains a note labeled "d♭". Measure 1₁₁ contains a note labeled "3.2". Measure 1₁₂ contains a note labeled "3.2". Measure 1₁₃ begins with a measure number 1₁₃ above the staff.

(2)

Musical score for Ennead 2, 15 measures. The score consists of four staves of music. Measure numbers 2₇ through 2₁₂ are indicated below the staff. Measure 2₉ contains a note labeled "c♯". Measure 2₁₀ contains a note labeled "d♭". Measure 2₁₁ contains a note labeled "3.2". Measure 2₁₂ contains a note labeled "3.2". Measure 2₁₃ begins with a measure number 2₁₃ above the staff.

(3)

Musical score for Ennead 3, 15 measures. The score consists of four staves of music. Measure numbers 3₇ through 3₁₂ are indicated below the staff. Measure 3₉ contains a note labeled "c". Measure 3₁₀ contains a note labeled "d". Measure 3₁₁ contains a note labeled "3.2". Measure 3₁₂ contains a note labeled "5.2". Measure 3₁₃ begins with a measure number 3₁₃ above the staff.

...Enneads...

(4)

Musical score for Ennead 4, featuring six staves of music. The score consists of 15 measures, divided into three groups of five measures each by vertical dashed lines. Measure numbers are placed below each group: 4_1 , 4^1 , 4_2 , 4^2 , 4_3 , 4^3 , 4_4 , 4^4 , 4_5 , 4^5 , 4_6 , and 4^6 . Measure 4_1 has a duration of 3.1, while 4^1 through 4^6 have a duration of 1.

(5)

Musical score for Ennead 5, featuring six staves of music. The score consists of 15 measures, divided into three groups of five measures each by vertical dashed lines. Measure numbers are placed below each group: 5_1 , 5^1 , 5_2 , 5^2 , 5_3 , 5^3 , 5_4 , 5^4 , 5_5 , 5^5 , 5_6 , and 5^6 . Measure 5_1 has a duration of 6, while 5^1 through 5^6 have a duration of 5.1.

(6)

Musical score for Ennead 6, featuring six staves of music. The score consists of 15 measures, divided into three groups of five measures each by vertical dashed lines. Measure numbers are placed below each group: 6_1 , 6^1 , 6_2 , 6^2 , 6_3 , 6^3 , 6_4 , 6^4 , 6_5 , 6^5 , 6_6 , and 6^6 . Measure 6_1 has a duration of 6, while 6^1 through 6^6 have a duration of 5.1.

...Enneads

(4)

15

4_7 4^7 4_8 4^8 4_9 4^9 4_{10} 4^{10} 4_{11} 4^{11} 4_{12} 4^{12}
 1 1 3.1 3.1 1 1 1

(5)

15

5_7 5^7 5_8 5^8 5_9 5^9 5_{10} 5^{10} 5_{11} 5^{11} 5_{12} 5^{12}
 5.1 5.1 6 6 5.1 5.1

(6)

15

6_7 6^7 6_8 6^8 6_9 6^9 6_{10} 6^{10} 6_{11} 6^{11} 6_{12} 6^{12}
 5.1 5.1 6 6 5.1 5.1

8.4

Odd Enneads Sorted by Rotations...

I

Section I contains six odd enneads, numbered 1 through 6, each consisting of nine measures. The score is in 12/8 time, indicated by a 12 over 8 time signature at the top right. The key signature changes for each ennead: 1 has one flat, 2 has two flats, 3 has one sharp, 4 has one sharp, 5 has one sharp, and 6 has one flat. Measure numbers are provided below each group of measures.

Ennead	Measure 1	Measure 2	Measure 3	Measure 4	Measure 5	Measure 6	Measure 7	Measure 8	Measure 9
1	1 ₁	2	1 ¹						
2			2 ₁	2	2 ¹				
3				3 ₁	4	3 ¹			

(5)

Continuation of Section I, showing the remaining three odd enneads (4, 5, and 6) in 12/8 time. The key signatures remain consistent with the previous section: 4 has one sharp, 5 has one sharp, and 6 has one flat. Measure numbers are provided below each group of measures.

Ennead	Measure 1	Measure 2	Measure 3	Measure 4	Measure 5	Measure 6	Measure 7	Measure 8	Measure 9
4	4 ₁	3.1	4 ¹						
5			5 ₁	6	5 ¹				
6				6 ₁	6	6 ¹			

II

Section II contains six odd enneads, numbered 1 through 6, each consisting of nine measures. The score is in 12/8 time, indicated by a 12 over 8 time signature at the top right. The key signature changes for each ennead: 1 has two sharps, 2 has one sharp, 3 has one sharp, 4 has two sharps, 5 has one sharp, and 6 has one sharp. Measure numbers are provided below each group of measures.

Ennead	Measure 1	Measure 2	Measure 3	Measure 4	Measure 5	Measure 6	Measure 7	Measure 8	Measure 9
1	1 ₃	3.2	1 ³						
2			2 ₃	3.2	2 ³				
3				3 ₃	5.2	3 ³			

8.5

...Odd Enneads Sorted by Rotations...

Staff 1 (Top):

- Circle 4: 4_3 , 1
- Circle 5: 4^3 , 5_3 , 5.1
- Circle 6: 5^3 , 6_3 , 5.1

Staff 2 (Middle):

- Circle 1: 1_5 , 2
- Circle 2: 1^5 , 2_5 , 2
- Circle 3: 2^5 , 3_5 , 4

Staff 3 (Bottom):

- Circle 4: 4_5 , 3.1
- Circle 5: 4^5 , 5_5 , 6
- Circle 6: 5^5 , 6_5 , 6

...Odd Enneads Sorted by Rotations...

IV

Musical notation for odd enneads 1, 2, and 3 in 4/4 time. The notation consists of three staves of nine measures each, separated by vertical bar lines. The first staff has a key signature of two sharps (F# major). The second staff has a key signature of one sharp (G major). The third staff has a key signature of no sharps or flats (C major). Measure numbers 1 through 9 are placed below each staff.

1₇ 1⁷ 2₇ 2⁷ 3₇ 3⁷
 3.2 3.2 5.2

Musical notation for odd enneads 4, 5, and 6 in 4/4 time. The notation consists of three staves of nine measures each, separated by vertical bar lines. The first staff has a key signature of one flat (B-flat major). The second staff has a key signature of no sharps or flats (C major). The third staff has a key signature of one sharp (G major). Measure numbers 1 through 9 are placed below each staff.

4₇ 4⁷ 5₇ 5⁷ 6₇ 6⁷
 1 5.1 5.1

V

Musical notation for odd enneads 1, 2, and 3 in 2/4 time. The notation consists of three staves of nine measures each, separated by vertical bar lines. The first staff has a key signature of one flat (B-flat major). The second staff has a key signature of no sharps or flats (C major). The third staff has a key signature of one sharp (G major). Measure numbers 1 through 9 are placed below each staff.

1₉ 1⁹ 2₉ 2⁹ 3₉ 3⁹
 2 2 4

8.7

...Odd Enneads Sorted by Rotations

(4) \textcircled{A}

(5) $\textcircled{B} \flat$

(6) $\textcircled{B} \flat$

4^9 4^9 5^9 6 5^9 6^9

3.1 6 6

VI

(1) $\textcircled{B} \flat$

(2) $\textcircled{B} \flat$

(3) $\textcircled{B} \flat$ $\textcircled{B} \flat$

1_{II} 1^{II} 2_{II} 2^{II} 3_{II} 3^{II}

3.2 3.2 2_{II} 2_{II} 3_{II} 5.2

(4) $\textcircled{B} \flat$

(5) \textcircled{A}

(6) $\textcircled{B} \flat$

4_{II} 4^{II} 5_{II} 5^{II} 6_{II} 6^{II}

1 5.1 5.1 6_{II} 6_{II}

Even Enneads Sorted by Rotations...

I

①

②

③

Musical score for Even Ennead 1 (1²). The score consists of three staves of music. The first staff has a key signature of two sharps. The second staff has a key signature of one sharp. The third staff has a key signature of one sharp. The music is divided into measures by vertical bar lines.

1² 1₂ 2² 2₂ 3² 3₂

④

⑤

⑥

Musical score for Even Enneads 4, 5, and 6. The score consists of three staves of music. The first staff has a key signature of one sharp. The second staff has a key signature of one sharp. The third staff has a key signature of one sharp. The music is divided into measures by vertical bar lines.

4² 4₂ 5² 5₂ 6² 6₂

II

①

②

③

Musical score for Even Enneads 1, 2, and 3 in section II. The score consists of three staves of music. The first staff has a key signature of one sharp. The second staff has a key signature of one sharp. The third staff has a key signature of one sharp. The music is divided into measures by vertical bar lines.

1⁴ 1₄ 2⁴ 2₄ 3⁴ 3₄

8.9

...Even Enneads Sorted by Rotations...

Staff 4:

- Measures 1-2: 4^4 , 4_4
- Measures 3-4: 5^4 , 5.1
- Measures 5-6: 6^4 , 5.1
- Section III: 6^4 , 6_4

Staff 5:

- Measures 1-2: 1^6 , 1_6
- Measures 3-4: 2^6 , 2_6
- Measures 5-6: 3^6 , 3_6

Staff 6:

- Measures 1-2: 4^6 , 4_6
- Measures 3-4: 5^6 , 5_6
- Measures 5-6: 6^6 , 6_6

Section III: 6^4 , 6_4

...Even Enneads Sorted by Rotations...

IV

1⁸ 1₈ 2⁸ 2₈ 3⁸ 3₈

3.2 3.2 5.2

4⁸ 4₈ 5⁸ 5₈ 6⁸ 6₈

1 5.1 5.1

V

1¹⁰ 1₁₀ 2¹⁰ 2₁₀ 3¹⁰ 3₁₀

2 2 4

...Even Enneads Sorted by Rotations

Musical score for Even Enneads 4, 5, and 6. The score consists of three staves of music. Staff 1 (top) contains measures 4¹⁰, 4₁₀, 5¹⁰, 5₁₀, 6¹⁰, and 6₁₀. Staff 2 (middle) contains measures 3.1, 6, and 6. Staff 3 (bottom) contains measures 3.1, 6, and 6. The score is labeled with circled numbers 4, 5, and 6 above the staves. The key signature is C major (no sharps or flats). Measure 4¹⁰ starts with a sharp on the first line. Measure 5¹⁰ starts with a flat on the second line. Measure 6¹⁰ starts with a flat on the third line.

VI

Musical score for Even Enneads 1, 2, and 3. The score consists of three staves of music. Staff 1 (top) contains measures 1¹², 1₁₂, 2¹², 2₁₂, 3¹², and 3₁₂. Staff 2 (middle) contains measures 3.2, 3.2, 3.2, 3.2, 5.2, and 5.2. Staff 3 (bottom) contains measures 3.2, 3.2, 3.2, 3.2, 5.2, and 5.2. The score is labeled with circled numbers 1, 2, and 3 above the staves. The key signature is C major (no sharps or flats). Measure 1¹² starts with a sharp on the first line. Measure 2¹² starts with a flat on the second line. Measure 3¹² starts with a flat on the third line.

Musical score for Even Enneads 4, 5, and 6. The score consists of three staves of music. Staff 1 (top) contains measures 4¹², 4₁₂, 5¹², 5₁₂, 6¹², and 6₁₂. Staff 2 (middle) contains measures 1, 5.1, 5.1, 5.1, 8.12, and 8.12. Staff 3 (bottom) contains measures 1, 5.1, 5.1, 5.1, 8.12, and 8.12. The score is labeled with circled numbers 4, 5, and 6 above the staves. The key signature is C major (no sharps or flats). Measure 4¹² starts with a flat on the second line. Measure 5¹² starts with a sharp on the first line. Measure 6¹² starts with a sharp on the first line.

8.12

Horizontal Enneads 1

(1)

(1)

▽ 2 1₁→ 1₂→ 1₅→ 1₆→ 1₉→ 1₁₀→ 1₁₁ 2

△ 3.2 1₃→ 1₄→ 1₇→ 1₈→ 1₁₁→ 1₁₂→ 1₁₃ 3.2

(2)

(2)

▽ 2 2₁→ 2₂→ 2₅→ 2₆→ 2₉→ 2₁₀→ 2₁₁ 2

△ 3.2 2₃→ 2₄→ 2₇→ 2₈→ 2₁₁→ 2₁₂→ 2₁₃ 3.2

(3)

(3)

▽ 4 3₁→ 3₂→ 3₅→ 3₆→ 3₉→ 3₁₀→ 3₁₁ 4

△ 5.2 3₃→ 3₄→ 3₇→ 3₈→ 3₁₁→ 3₁₂→ 3₁₃ 5.2

(4)

(4)

▽ 3.1 4₁→ 4₂→ 4₅→ 4₆→ 4₉→ 4₁₀→ 4₁₁ 3.1

△ 1 4₃→ 4₄→ 4₇→ 4₈→ 4₁₁→ 4₁₂→ 4₁₃ 1

(5)

(5)

▽ 6 5₁→ 5₂→ 5₅→ 5₆→ 5₉→ 5₁₀→ 5₁₁ 6

△ 5.1 5₃→ 5₄→ 5₇→ 5₈→ 5₁₁→ 5₁₂→ 5₁₃ 5.1

(6)

(6)

▽ 6 6₁→ 6₂→ 6₅→ 6₆→ 6₉→ 6₁₀→ 6₁₁ 6

△ 5.1 6₃→ 6₄→ 6₇→ 6₈→ 6₁₁→ 6₁₂→ 6₁₃ 5.1

Horizontal Enneads 2

(1)

▽  ▽

△  △

(2)

▽  ▽

△  △

(3)

▽  ▽

△  △

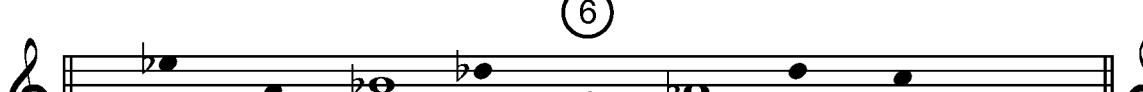
(4)

▽  ▽

△  △

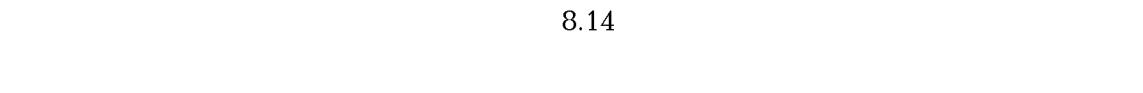
(5)

▽  ▽

△  △

(6)

▽  ▽

△  △

Dodecads...

(1)

15

1₁ 1¹ 1₂ 1² 1₃ 1³ 1₄ 1⁴ 1₅ 1⁵ 1₆ 1⁶

(2)

15

2₁ 2¹ 2₂ 2² 2₃ 2³ 2₄ 2⁴ 2₅ 2⁵ 2₆ 2⁶

(3)

15

3₁ 3¹ 3₂ 3² 3₃ 3³ 3₄ 3⁴ 3₅ 3⁵ 3₆ 3⁶

...Dodecads...

(1)

Musical staff showing 12 dodecad chords labeled 1₇ to 1₁₂. The staff has 12 vertical columns of notes. The first column (1₇) starts with a bass note and includes a sharp sign. Subsequent columns show various note patterns, some with sharps and flats. The staff concludes with a bass note and a double flat sign.

1₇ 1⁷ 1₈ 1⁸ 1₉ 1⁹ 1₁₀ 1¹⁰ 1₁₁ 1¹¹ 1₁₂ 1¹²

(2)

Musical staff showing 12 dodecad chords labeled 2₇ to 2₁₂. The staff has 12 vertical columns of notes. The first column (2₇) starts with a bass note and includes a sharp sign. Subsequent columns show various note patterns, some with sharps and flats. The staff concludes with a bass note and a sharp sign.

2₇ 2⁷ 2₈ 2⁸ 2₉ 2⁹ 2₁₀ 2¹⁰ 2₁₁ 2¹¹ 2₁₂ 2¹²

(3)

Musical staff showing 12 dodecad chords labeled 3₇ to 3₁₂. The staff has 12 vertical columns of notes. The first column (3₇) starts with a bass note and includes a double flat sign. Subsequent columns show various note patterns, some with sharps and flats. The staff concludes with a bass note and a sharp sign.

3₇ 3⁷ 3₈ 3⁸ 3₉ 3⁹ 3₁₀ 3¹⁰ 3₁₁ 3¹¹ 3₁₂ 3¹²

...Dodecads...

Musical score for dodecad 4, page 15. The score consists of three staves (Treble, Alto, Bass) with 12 measures. Measure 4₁ through 4₆ are shown. The key signature is one flat. Measures 4₁ through 4₅ have a common time signature, while measure 4₆ has a different time signature. Measures 4₁ through 4₅ are grouped by vertical dashed lines. Measure 4₆ is preceded by a circled number 4.

Musical score for dodecad 5, page 15. The score consists of three staves (Treble, Alto, Bass) with 12 measures. Measure 5₁ through 5₆ are shown. The key signature changes frequently, indicated by various sharps and flats. Measures 5₁ through 5₅ have a common time signature, while measure 5₆ has a different time signature. Measures 5₁ through 5₅ are grouped by vertical dashed lines. Measure 5₆ is preceded by a circled number 5.

Musical score for dodecad 6, page 15. The score consists of three staves (Treble, Alto, Bass) with 12 measures. Measure 6₁ through 6₆ are shown. The key signature changes frequently, indicated by various sharps and flats. Measures 6₁ through 6₅ have a common time signature, while measure 6₆ has a different time signature. Measures 6₁ through 6₅ are grouped by vertical dashed lines. Measure 6₆ is preceded by a circled number 6.

...Dodecads

(4)

Musical staff showing 12 dodecadic patterns labeled 4₁ through 4₁₂. The staff has 12 vertical columns of notes. The notes are represented by black dots on a grid. The patterns are divided into four groups by vertical dashed lines: group 1 (4₁-4₄) uses mostly black dots; group 2 (4₅-4₈) includes some white dots; group 3 (4₉-4₁₀) includes some black dots with stems; group 4 (4₁₁-4₁₂) includes some white dots with stems. The notes are grouped into pairs or sets of three in each column.

(5)

Musical staff showing 12 dodecadic patterns labeled 5₁ through 5₁₂. The staff has 12 vertical columns of notes. The patterns are divided into four groups by vertical dashed lines: group 1 (5₁-5₄) uses mostly black dots; group 2 (5₅-5₈) includes some white dots; group 3 (5₉-5₁₀) includes some black dots with stems; group 4 (5₁₁-5₁₂) includes some white dots with stems. The notes are grouped into pairs or sets of three in each column.

(6)

Musical staff showing 12 dodecadic patterns labeled 6₁ through 6₁₂. The staff has 12 vertical columns of notes. The patterns are divided into four groups by vertical dashed lines: group 1 (6₁-6₄) uses mostly black dots; group 2 (6₅-6₈) includes some white dots; group 3 (6₉-6₁₀) includes some black dots with stems; group 4 (6₁₁-6₁₂) includes some white dots with stems. The notes are grouped into pairs or sets of three in each column.

Horizontal Dodecads 1

(1)

Musical staff for horizontal dodecad 1. The staff consists of five horizontal lines and four spaces. Notes are placed on the lines and spaces, with some being filled circles and others hollow circles with a dot. Below the staff, numbered arrows indicate specific notes: 1₁→, 1₂→, 1₃→, 1₄→, 1₅→, 1₆→, 1₇→, 1₈→, 1₉→, 1₁₀→, 1₁₁→, 1₁₂→, ←1², ←1³, ←1⁴, ←1⁵, ←1⁶, ←1⁷, ←1⁸, ←1⁹, ←1¹⁰, ←1¹¹, ←1¹², ←1¹.

(2)

Musical staff for horizontal dodecad 2. The staff consists of five horizontal lines and four spaces. Notes are placed on the lines and spaces, with some being filled circles and others hollow circles with a dot. Below the staff, numbered arrows indicate specific notes: 2₁→, 2₂→, 2₃→, 2₄→, 2₅→, 2₆→, 2₇→, 2₈→, 2₉→, 2₁₀→, 2₁₁→, 2₁₂→, ←2², ←2³, ←2⁴, ←2⁵, ←2⁶, ←2⁷, ←2⁸, ←2⁹, ←2¹⁰, ←2¹¹, ←2¹², ←2¹.

(3)

Musical staff for horizontal dodecad 3. The staff consists of five horizontal lines and four spaces. Notes are placed on the lines and spaces, with some being filled circles and others hollow circles with a dot. Below the staff, numbered arrows indicate specific notes: 3₁→, 3₂→, 3₃→, 3₄→, 3₅→, 3₆→, 3₇→, 3₈→, 3₉→, 3₁₀→, 3₁₁→, 3₁₂→, ←3², ←3³, ←3⁴, ←3⁵, ←3⁶, ←3⁷, ←3⁸, ←3⁹, ←3¹⁰, ←3¹¹, ←3¹², ←3¹.

(4)

Musical staff for horizontal dodecad 4. The staff consists of five horizontal lines and four spaces. Notes are placed on the lines and spaces, with some being filled circles and others hollow circles with a dot. Below the staff, numbered arrows indicate specific notes: 4₁→, 4₂→, 4₃→, 4₄→, 4₅→, 4₆→, 4₇→, 4₈→, 4₉→, 4₁₀→, 4₁₁→, 4₁₂→, ←4², ←4³, ←4⁴, ←4⁵, ←4⁶, ←4⁷, ←4⁸, ←4⁹, ←4¹⁰, ←4¹¹, ←4¹², ←4¹.

(5)

Musical staff for horizontal dodecad 5. The staff consists of five horizontal lines and four spaces. Notes are placed on the lines and spaces, with some being filled circles and others hollow circles with a dot. Below the staff, numbered arrows indicate specific notes: 5₁→, 5₂→, 5₃→, 5₄→, 5₅→, 5₆→, 5₇→, 5₈→, 5₉→, 5₁₀→, 5₁₁→, 5₁₂→, ←5², ←5³, ←5⁴, ←5⁵, ←5⁶, ←5⁷, ←5⁸, ←5⁹, ←5¹⁰, ←5¹¹, ←5¹², ←5¹.

(6)

Musical staff for horizontal dodecad 6. The staff consists of five horizontal lines and four spaces. Notes are placed on the lines and spaces, with some being filled circles and others hollow circles with a dot. Below the staff, numbered arrows indicate specific notes: 6₁→, 6₂→, 6₃→, 6₄→, 6₅→, 6₆→, 6₇→, 6₈→, 6₉→, 6₁₀→, 6₁₁→, 6₁₂→, ←6², ←6³, ←6⁴, ←6⁵, ←6⁶, ←6⁷, ←6⁸, ←6⁹, ←6¹⁰, ←6¹¹, ←6¹², ←6¹.

Horizontal Dodecads 2

(1)



Musical staff for horizontal dodecad 1. The notes are: ♭, ♯, ♭, ♯, ♭, ♯, ♭, ♯, ♭, ♯, ♭, ♯. The corresponding pitch labels below the staff are: $\leftarrow 1_{12}$, $\leftarrow 1_{11}$, $\leftarrow 1_{10}$, $\leftarrow 1_9$, $\leftarrow 1_8$, $\leftarrow 1_7$, $\leftarrow 1_6$, $\leftarrow 1_5$, $\leftarrow 1_4$, $\leftarrow 1_3$, $\leftarrow 1_2$, $\leftarrow 1_1$. Below these are the note numbers: $1^1 \rightarrow$, $1^{12} \rightarrow$, $1^{11} \rightarrow$, $1^{10} \rightarrow$, $1^9 \rightarrow$, $1^8 \rightarrow$, $1^7 \rightarrow$, $1^6 \rightarrow$, $1^5 \rightarrow$, $1^4 \rightarrow$, $1^3 \rightarrow$, $1^2 \rightarrow$.

(2)



Musical staff for horizontal dodecad 2. The notes are: ♭, ♭, ♭, ♯, ♭, ♭, ♭, ♯, ♭, ♭, ♭, ♭. The corresponding pitch labels below the staff are: $\leftarrow 2_{12}$, $\leftarrow 2_{11}$, $\leftarrow 2_{10}$, $\leftarrow 2_9$, $\leftarrow 2_8$, $\leftarrow 2_7$, $\leftarrow 2_6$, $\leftarrow 2_5$, $\leftarrow 2_4$, $\leftarrow 2_3$, $\leftarrow 2_2$, $\leftarrow 2_1$. Below these are the note numbers: $2^1 \rightarrow$, $2^{12} \rightarrow$, $2^{11} \rightarrow$, $2^{10} \rightarrow$, $2^9 \rightarrow$, $2^8 \rightarrow$, $2^7 \rightarrow$, $2^6 \rightarrow$, $2^5 \rightarrow$, $2^4 \rightarrow$, $2^3 \rightarrow$, $2^2 \rightarrow$.

(3)



Musical staff for horizontal dodecad 3. The notes are: ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭. The corresponding pitch labels below the staff are: $\leftarrow 3_{12}$, $\leftarrow 3_{11}$, $\leftarrow 3_{10}$, $\leftarrow 3_9$, $\leftarrow 3_8$, $\leftarrow 3_7$, $\leftarrow 3_6$, $\leftarrow 3_5$, $\leftarrow 3_4$, $\leftarrow 3_3$, $\leftarrow 3_2$, $\leftarrow 3_1$. Below these are the note numbers: $3^1 \rightarrow$, $3^{12} \rightarrow$, $3^{11} \rightarrow$, $3^{10} \rightarrow$, $3^9 \rightarrow$, $3^8 \rightarrow$, $3^7 \rightarrow$, $3^6 \rightarrow$, $3^5 \rightarrow$, $3^4 \rightarrow$, $3^3 \rightarrow$, $3^2 \rightarrow$.

(4)



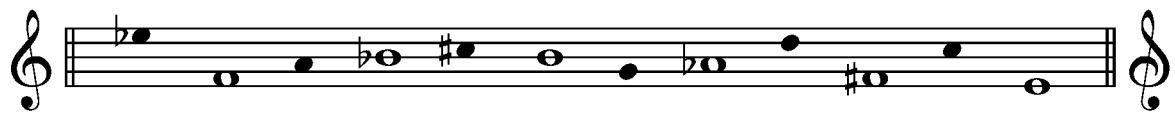
Musical staff for horizontal dodecad 4. The notes are: ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭. The corresponding pitch labels below the staff are: $\leftarrow 4_{12}$, $\leftarrow 4_{11}$, $\leftarrow 4_{10}$, $\leftarrow 4_9$, $\leftarrow 4_8$, $\leftarrow 4_7$, $\leftarrow 4_6$, $\leftarrow 4_5$, $\leftarrow 4_4$, $\leftarrow 4_3$, $\leftarrow 4_2$, $\leftarrow 4_1$. Below these are the note numbers: $4^1 \rightarrow$, $4^{12} \rightarrow$, $4^{11} \rightarrow$, $4^{10} \rightarrow$, $4^9 \rightarrow$, $4^8 \rightarrow$, $4^7 \rightarrow$, $4^6 \rightarrow$, $4^5 \rightarrow$, $4^4 \rightarrow$, $4^3 \rightarrow$, $4^2 \rightarrow$.

(5)



Musical staff for horizontal dodecad 5. The notes are: ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭. The corresponding pitch labels below the staff are: $\leftarrow 5_{12}$, $\leftarrow 5_{11}$, $\leftarrow 5_{10}$, $\leftarrow 5_9$, $\leftarrow 5_8$, $\leftarrow 5_7$, $\leftarrow 5_6$, $\leftarrow 5_5$, $\leftarrow 5_4$, $\leftarrow 5_3$, $\leftarrow 5_2$, $\leftarrow 5_1$. Below these are the note numbers: $5^1 \rightarrow$, $5^{12} \rightarrow$, $5^{11} \rightarrow$, $5^{10} \rightarrow$, $5^9 \rightarrow$, $5^8 \rightarrow$, $5^7 \rightarrow$, $5^6 \rightarrow$, $5^5 \rightarrow$, $5^4 \rightarrow$, $5^3 \rightarrow$, $5^2 \rightarrow$.

(6)



Musical staff for horizontal dodecad 6. The notes are: ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭, ♭. The corresponding pitch labels below the staff are: $\leftarrow 6_{12}$, $\leftarrow 6_{11}$, $\leftarrow 6_{10}$, $\leftarrow 6_9$, $\leftarrow 6_8$, $\leftarrow 6_7$, $\leftarrow 6_6$, $\leftarrow 6_5$, $\leftarrow 6_4$, $\leftarrow 6_3$, $\leftarrow 6_2$, $\leftarrow 6_1$. Below these are the note numbers: $6^1 \rightarrow$, $6^{12} \rightarrow$, $6^{11} \rightarrow$, $6^{10} \rightarrow$, $6^9 \rightarrow$, $6^8 \rightarrow$, $6^7 \rightarrow$, $6^6 \rightarrow$, $6^5 \rightarrow$, $6^4 \rightarrow$, $6^3 \rightarrow$, $6^2 \rightarrow$.

Opposite Vertices

(Horizontal Dodecads in 2)

(1) $\xrightarrow{\xleftarrow{8}}$

1₂⁴₂ 1₄₄ 1₆³₆(4₃5₅) 1²₄₂ 1⁴₄ 1⁶3₆(4³5⁵) 1₂⁴₂ 1₄₄ 1₆³₆(4₃5₅) 1²₄₂ 1⁴₄ 1⁶3₆(4³5⁵)

1₁2₁4₁ 1₃(4₆) 1₅4⁵ 1¹2¹4¹ 1³(4⁶) 1⁵4₅ 1₁2₁4₁ 1₃(4₆) 1₅4⁵ 1¹2¹4¹ 1³(4⁶) 1⁵4₅

(2) $\xrightarrow{\xleftarrow{8}}$

2₂ 2₃ 2₄ 2₅ 2₆ 2² 2³ 2⁴ 2⁵ 2⁶ 2₂ 2₃ 2₄ 2₅ 2₆ 2² 2³ 2⁴ 2⁵ 2⁶

1₁2₁4₁ 1¹2¹4¹ 1₁2₁4₁ 1¹2¹4¹

(3) $\xrightarrow{\xleftarrow{8}}$

3₂5₆ 3₄5⁴ 1⁶3₆(4³5⁵) 3²5⁶ 3⁴5₄ 1₆3₆(4₃5₅) 3₂5₆ 3₄5⁴ 1⁶3₆(4³5⁵) 3²5⁶ 3⁴5₄ 1⁶3₆(4₃5₅)

3₁5₁ 3₃(5²) 3₅5₃6₃ 3¹5¹ 3³(5₂) 3⁵5³6³ 3₁5₁ 3₃(5²) 3₅5₃6₃ 3¹5¹ 3³(5₂) 3⁵5³6³

(4) $\xrightarrow{\xleftarrow{8}}$

1²₄₂ 1₄₄ (1₃)4₆ 1₂⁴₂ 1⁴₄ 1⁽³⁾4⁶ 1²₄₂ 1₄₄ (1₃)4₆ 1₂⁴₂ 1⁴₄ 1⁽³⁾4⁶

1₁2₁4₁ (1₆3⁶)4₃5₅ 1⁵4₅ 1¹2¹4¹ (1⁶3₆)4³5⁵ 1⁵4₅ 1₁2₁4₁ (1₆3⁶)4₃5₅ 1⁵4₅ 1¹2¹4¹ (1⁶3₆)4³5⁵ 1⁵4₅

(5) $\xrightarrow{\xleftarrow{8}}$

(3³)5₂ 3⁴5₄ 3₂5₆ (3₃)5² 3⁴5₄ 3²5⁶ (3³)5₂ 3⁴5₄ 3₂5₆ (3₃)5² 3⁴5₄ 3²5⁶

3₁5₁ 3₅5₃6₃ (1₆3⁶)4₃5₅ 3¹5¹ 3⁵5³6³ (1⁶3₆)4³5⁵ 3₁5₁ 3₅5₃6₃ (1₆3⁶)4₃5₅ 3¹5¹ 3⁵5³6³ (1⁶3₆)4³5⁵

(6) $\xrightarrow{\xleftarrow{8}}$

6₁ 6₂ 6₄ 6₅ 6₆ 6¹ 6² 6⁴ 6⁵ 6⁶ 6₁ 6₂ 6₄ 6₅ 6₆ 6¹ 6² 6⁴ 6⁵ 6⁶

3₅5₃6₃ 3⁵5³6³ 3₅5₃6₃ 3⁵5³6³

Horizontal Opposite Vertices

(1)

1₁2₁4₁ → 1₂4² → 1₃(4₆) → 1₄4₄ → 1₅4⁵ → 1₆3⁶(4₃5₅) → 1¹2¹4¹ → 1²4₂ → 1³(4⁶) → 1⁴4⁴ → 1⁵4₅ → 1⁶3₆(4³5⁵) →
 ←1¹2¹4¹ ←1²4₂ ←1³(4⁶) ←1⁴4⁴ ←1⁵4₅ ←1⁶3₆(4³5⁵) ←1₁2₁4₁ ←1₂4² ←1₃(4₆) ←1₄4₄ ←1₅4⁵ ←1₆3₆(4₃5₅)

(2)

1₁2₁4₁ → 2₂ → 2₃ → 2₄ → 2₅ → 2₆ → 1¹2¹4¹ → 2² → 2³ → 2⁴ → 2⁵ → 2⁶ →
 ←1¹2¹4¹ ←2² ←2³ ←2⁴ ←2⁵ ←2⁶ ←1₁2₁4₁ ←2₂ ←2₃ ←2₄ ←2₅ ←2₆

(3)

3₁5₁ → 3₂5₆ → 3₃(5²) → 3₄5⁴ → 3₅5₃6₃ → 1⁶3₆(4³5⁵) → 3¹5₁ → 3²5⁶ → 3³(5₂) → 3⁴5₄ → 3⁵5³6³ → 1₆3₆(4₃5₅) →
 ←3¹5¹ ←3²5⁶ ←3³(5₂) ←3⁴5₄ ←3⁵5³6³ ←1₆3₆(4₃5₅) ←3₁5₁ ←3₂5₆ ←3₃(5²) ←3⁴5₄ ←3₅5₃6₃ ←1⁶3₆(4³5⁵)

(4)

1₁2₁4₁ → 1²4₂ → (1₆3⁶)4₃5₅ → 1₄4₄ → 1⁵4₅ → (1₃)4₆ → 1¹2¹4¹ → 1₂4² → (1⁶3₆)4³5⁵ → 1⁴4⁴ → 1₅4⁵ → (1³)4⁶ →
 ←1¹2¹4¹ ←1₂4² ←(1₆3₆)4³5⁵ ←1⁴4⁴ ←1₅4⁵ ←(1³)4⁶ ←1₁2₁4₁ ←1₂4² ←(1₆3₆)4₃5₅ ←1₄4₄ ←1⁵4₅ ←(1₃)4₆

(5)

3₁5₁ → (3³)5₂ → 3₅5₃6₃ → 3⁴5₄ → (1₆3⁶)4₃5₅ → 3₂5₆ → 3¹5₁ → (3₃)5² → 3⁵5³6³ → 3⁴5₄ → (1⁶3₆)4³5⁵ → 3²5⁶ →
 ←3¹5¹ ←(3₃)5² ←3⁵5³6³ ←3⁴5₄ ←(1₆3₆)4³5⁵ ←3²5₆ ←3₁5₁ ←(3₃)5₂ ←3₅5₃6₃ ←3⁴5₄ ←(1₆3₆)4₃5₅ ←3₂5₆

(6)

6₁ → 6₂ → 3₅5₃6₃ → 6₄ → 6₅ → 6₆ → 6¹ → 6² → 3⁵5³6³ → 6⁴ → 6⁵ → 6⁶ →
 ←6¹ ←6² ←3⁵5³6³ ←6⁴ ←6⁵ ←6⁶ ←6₁ ←6₂ ←3₅5₃6₃ ←6₄ ←6₅ ←6₆

